# 2025 SCSU MATH CONTEST
## $9^{th}$ and $10^{th}$ GRADE

DIRECTIONS: Select the BEST completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. Welcome to the 2025 SCSU Math Contest – it's a perfect time to be here!  In honor of the occasion, find the sum of all perfect square factors of 2025.
   A. 2141      B. 2285      C. 2343      D. 2366      E. None of these

2. How many distinguishable rearrangements of the letters in the word CONTEST have both the vowels first? For instance, OETCNTS is one such arrangement, but OTETSNC is not.
   A. 60      B. 120      C. 240      D. 720      E. None of these

3. A polygon is convex if any two points in the polygon can be connected by a line segment inside the polygon. Suppose exactly three of the interior angles of a convex polygon are obtuse. What is the maximum number of sides of such a polygon?
   A. 4      B. 5      C. 6      D. 7      E. 8

4. Janet used a 3% iodine solution and a 20% iodine solution to make an 85-ounce solution that was 19% iodine. How many ounces of the 3% solution did she use?
   A. 5      B. 6      C. 7      D. 8      E. None of these

5. Electrical resistance in a wire is directly proportional to its length and inversely proportional to the square of its diameter.  If a 10-centimeter long wire with a diameter of 2 centimeters has resistance 600 ohms, find the resistance of a 15-centimeter long wire with a diameter of 5 centimeters.
   A. 30 ohms      B. 60 ohms      C. 120 ohms      D. 144 ohms      E. None of these

6. In a survey of 200 high school students, the following is obtained about the courses they are taking:
   - 81 are enrolled in Spanish
   - 88 are enrolled in geometry
   - 96 are enrolled in history
   - 22 are enrolled in both Spanish and history
   - 28 are enrolled in both Spanish and geometry
   - 38 are enrolled in both geometry and history
   - 20 are not enrolled in any of these three courses.

   How many of the 200 students are enrolled in all three courses?
   A. 3      B. 5      C. 7      D. 8      E. None of these

7. In the figure (below left), a rectangle intersects a circle as shown where:
   $AB = 4, BC = 5,$ and $DE = 3$. Then, $EF$ equals
   A. 6      B. 7      C. $\frac{20}{3}$      D. 8      E. 9
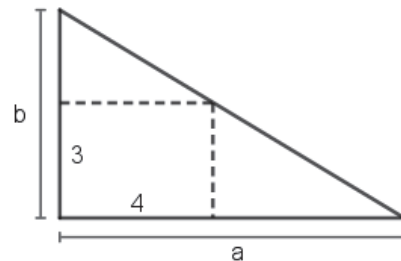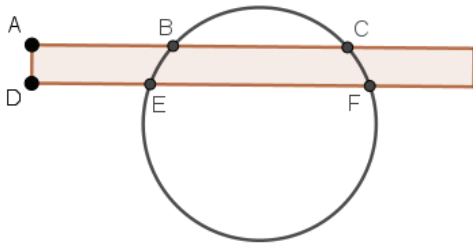


8. Consider the figure (above right), which is not necessarily drawn to scale.
   Find $(a - 4)(b - 3)$.
   A. 3      B. 6      C. 8      D. 12      E. None of these

9. When the decimal number $0.0\overline{12}$ is converted to a fraction in lowest terms, what is the sum of the digits in the numerator and denominator?

   A. 4          B. 11          C. 14          D. 21          E. None of these

10. Find the number of solutions of the system of equations below.
$$x = x^2 + y^2$$
$$y = 2xy$$

    A. 1          B. 2          C. 3          D. 4          E. Infinitely many

11. A 15-foot wide inground pool is filled with water. The shallow end is 3 feet deep and slopes linearly to the deepest end, which is 10 feet deep. The length of the pool is 30 feet. In gallons, what is the volume of water in the pool? Round your answer to the nearest gallon. Fact: 1 gallon is 231 cubic inches.

    A. 10,099          B. 11,782          C. 21,881          D. 33,662          E. None of these

12. The heights of four athletes on a team are 135 cm, 160 cm, 170 cm, and 175 cm. Lukas joins the team. On the new team of five athletes, the mode height of the players is equal to the median height which is equal to the mean height. How tall is Lukas?

    A. 135 cm          B. 160 cm          C. 165 cm          D. 170 cm          E. None of these

13. ABCDE is an even number that is less than 25,000. Given that all digits are distinct, none of them are 3 or 6, and that 4 times ABCDE is EDCBA, find A+B+C+D+E.

    A. 19          B. 21          C. 24          D. 27          E. None of these

14. The manager of a floral shop ordered two different kinds of marigolds. The yellow marigolds cost $1.00 per plant and the gold cost $1.26 per plant. She bought 50 more gold plants than yellow and spent a total of $402 for these plants. How many yellow marigolds did she buy?

    A. 150          B. 160          C. 164          D. 170          E. None of these

15. There are two natural ways of inscribing a square into a given isosceles right triangle. If it is done as in Figure 1 below (partitioning it into a square and two smaller isosceles triangles), then one finds that the area of the square is 441 in². What is the area (in in²) of the square inscribed in the same triangle as shown in Figure 2 below?
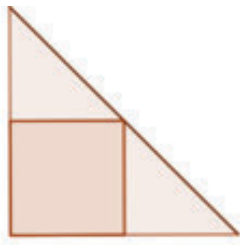
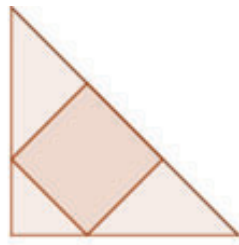    A. 378          B. 392          C. 400          D. 441          E. 484

Figure 1          Figure 2

16. Terry has a jar that contains $1.00 worth of dimes and $1.00 worth of quarters and no other coins. If he randomly removes two coins from the jar, what is the probability that both coins are dimes?

    A. $\frac{1}{90}$          B. $\frac{1}{5}$          C. $\frac{1}{4}$          D. $\frac{25}{49}$          E. $\frac{45}{91}$

17. Determine ones digit of the number $7^{2025}$.

    A. 1          B. 3          C. 5          D. 7          E. 9

18. In the figure (below left), a shaded triangle appears in the $3 \times 3$ grid shown. Which of the following is closest to the percentage of the square region shaded?

    A. 25%         B. 27%         C. 30%         D. 33%         E. 35%
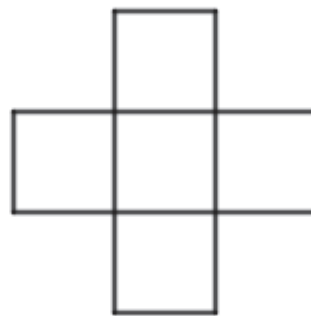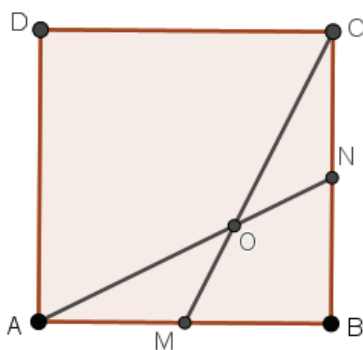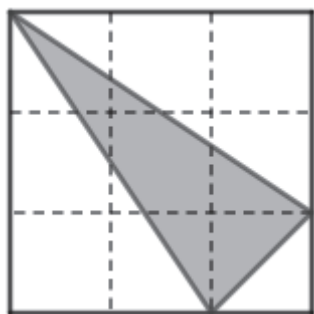
19. The square $ABCD$ (above center) has side length 1. Let $M$ and $N$ be the midpoints of $\overline{AB}$ and $\overline{BC}$, respectively. $\overline{AN}$ and $\overline{CM}$ intersect at point $O$. Find the area of quadrilateral $AOCD$.

    A. $\frac{5}{6}$         B. $\frac{3}{4}$         C. $\frac{2}{3}$         D. $\frac{\sqrt{3}}{2}$         E. $\frac{\sqrt{3}-1}{2}$

20. Five *different* integers from 1 to 6 are selected and one integer is placed in each of the five squares shown in the grid (above right). The integers are placed so that the sum of the three integers in the vertical column is 7, and the sum of the three integers in the horizontal row is 11. Which integer does NOT appear in any square?

    A. 1         B. 2         C. 3         D. 4         E. 5

21. In triangle $ABC$ (below left), $AC = BC$, $AD = AF$, and $BE = BF$. If the measure of $\angle C = 80°$, find the measure of $\angle DFE$.

    A. 30°         B. 40°         C. 50°         D. 65°         E. None of these
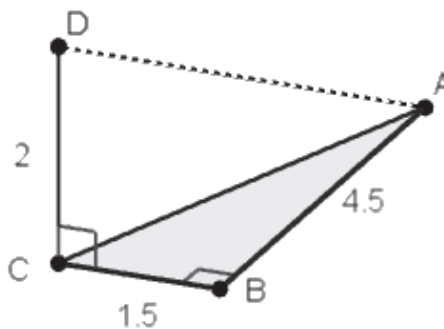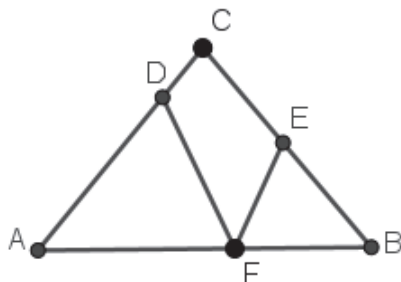
22. In the figure (above right), points $A, B$, and $C$ are in the same plane and point $D$ is above that plane. $\angle ABC$ and $\angle BCD$ are right angles and the distances between points are given. What is the distance between points $A$ and $D$? Express the answer to the nearest hundredth.

    A. 2.50         B. 4.74         C. 4.92         D. 5.15         E. None of these

23. The positive integer $N$ has exactly 8 positive divisors including 1 and $N$. Two of the divisors are 10 and 15. What is the sum of all 8 divisors of $N$?

    A. 36         B. 42         C. 66         D. 70         E. 72

24. In a particular lottery, four different numbers from 1 through 50 are randomly selected, then one number from 51 to 75 is randomly selected. Matching the four numbers less than or equal to 50 (without regard to order) <u>and</u> the number greater than 50 results in winning the jackpot. What is the smallest number of tickets you could buy which would guarantee that you win the jackpot?

    A. 1,250         B. 5,757,500         C. 17,259,390         D. 138,180,000         E. 2,071,126,800

25. When $\left(x + \frac{2}{x^2}\right)^8$ is expanded and like powers of $x$ are collected, what is the coefficient of the $x^5$ term?
    A. 12          B. 14          C. 16          D. 18          E. 20

26. Find, to the nearest tenth, the area of the circle given by the equation below:
$$x^2 + y^2 - 2(y + 2x) = 31$$
    A. 17.5          B. 18.8          C. 97.4          D. 113.1          E. None of these

27. In the figure (below), $\overline{BD}$ bisects $\angle ABC$, $\overline{CD}$ bisects $\angle ACB$ and $\overline{EF} \parallel \overline{BC}$. If $AB = 12, AC = 18$, and $BC = 24$, then find the perimeter of triangle $AEF$.
    A. 30          B. 33          C. 36          D. 39          E. 42
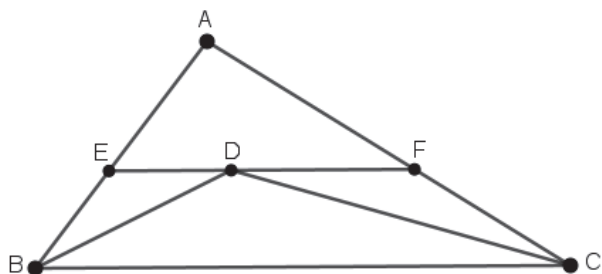


28. Given that $y = 4^x + 4^x$, which of the following is an expression for $y^4$?
    A. $256^x$          B. $16^{x+1}$          C. $4^{4x}$          D. $64^{x+1}$          E. $16^{2x+1}$

29. The first term in a sequence of whole numbers is 10. New terms of the sequence are generated by the following rules: If the current number is even, the next number is half the current number. If the current number is odd, the next number is calculated by multiplying the current number by 3 and adding 1. What is the 2,025th number in the sequence?
    A. 1          B. 2          C. 4          D. 16          E. None of these

30. If $x^2 + y^2 = 64$, what is the largest possible value of $3x + 4y$?
    A. 24          B. 32          C. 36          D. 38          E. 40

31. Suppose that $m$ is a positive integer. If $a_n$ is defined by $a_n = m + n^2$ for $n = 1, 2, 3, \ldots$, then what is the greatest common divisor of $a_{2m}$ and $a_{2m+1}$?
    A. $2m$          B. $2m + 1$          C. $4m$          D. 6          E. $4m + 1$

32. * Cyril owns more cats than Abdul and more dogs than Brad. Abdul owns more dogs than Cyril and fewer cats than Brad. Which of the statements *must* be true?
    A. Cyril owns the most cats.          B. Abdul owns the most dogs.          C. Brad owns the fewest cats.
    D. Brad owns the most cats.          E. Cyril owns more cats than Brad owns dogs.

33. Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{99}+\frac{1}{100}}{\frac{1}{99}+\frac{2}{98}+\frac{3}{97}+\cdots+\frac{98}{2}+\frac{99}{1}}$
    A. $\frac{1}{100}$          B. $\frac{1}{99}$          C. 1          D. 2          E. None of these

* This is a different version of #32 than was on the exam on the day of the contest. Previously, the question featured two correct answers and a misspelling. Additionally, an incorrect answer was changed to reduce confusion.