DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

1. Find the value of this expression: $\dfrac{2^{2019} + 2^{2017}}{2^{2018} + 2^{2016}}$

   A. $\dfrac{5}{3}$ B. 2   C. $\dfrac{5}{2}$ D. 3   E. 4

2. The sales of a business were 3 million dollars more the second year than the first, and sales for the third year were double the sales for the second year. If sales for the third year were 38 million dollars, what were sales, in millions of dollars, for the first year?

   A. 16   B. 17.5   C. 20.58   D. 22   E. 35

3. Mapquest is displaying a 100m X 100m land area on to a square region of the screen with pixel coordinates ranging from (760, 760) (top left corner) to (820, 820) (bottom right corner). A street corner appears at pixel location (800, 780).
   What is the distance, in meters, between this street corner and the center of the land area displayed?

   A. $\dfrac{25\sqrt{2}}{3}$ B. $10\sqrt{2}$ C. $\dfrac{50}{3}$ D. 20   E. $\dfrac{50\sqrt{2}}{3}$

4. 197,000,19X is a prime number. What is the value of X?

   A. 1   B. 3   C. 5   D. 7   E. 9

5. Find the difference between the sum of the even positive integers less than 1001 and the sum of the odd positive integers less than 1001.

   A. 0   B. 50   C. 250   D. 500   E. 1000

Use the diagrams below to answer questions 6 and 7.
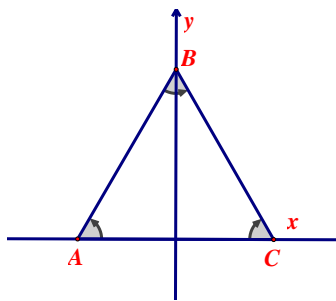
QUESTION 6   QUESTION 7
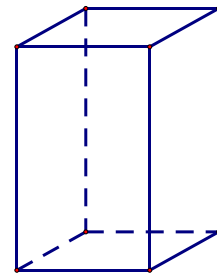
6. Triangle *ABC* shown in the diagram is an equilateral triangle.
   What is the slope of the line that passes through points *B* and *C*?

   A. -2 B. $-\sqrt{3}$ C. $-\dfrac{1}{2}$ D. $\sqrt{3}$ E. 2

7. Which of the following best approximates the surface area of the rectangular prism shown, in cm², if the base is a square with an area of 50 cm² and the height is 12 cm?

   A. 440   B. 526   C. 650   D. 1215   E. 2400

8. In triangle ABC, $m\angle B$ is equal to the square of $m\angle A$, and $m\angle C$ is 7 times as large as $m\angle A$.
   What is the measure, in degrees, of the largest angle of this triangle?
   A. 18   B. 70   C. 100   D. 126   E. 144

9. Consider the set of all five-digit integers such that the product of its five digits is 7!.
   What is the sum of the five digits in the largest element of the set?
   A. 27   B. 28   C. 29   D. 30   E. 31

10. A three-digit natural number is divisible by 5 but not by 10. The hundreds digit is odd and the tens digit is twice the hundreds digit. What is the sum of all three-digit numbers that meet these conditions?
   A. 490   B. 540   C. 560   D. 600   E. 660

11. One square can be constructed using all five shapes shown below: one square, one parallelogram and three right triangles with the measurements shown. What is the perimeter of the square that is formed?
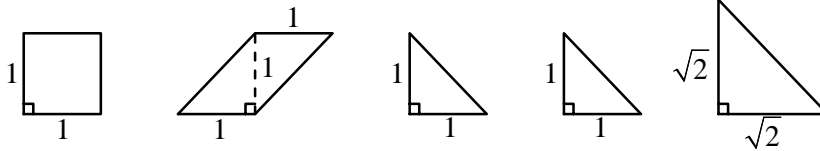
    A. $4\sqrt{2}$ B. $4+2\sqrt{2}$ C. $2+4\sqrt{2}$ D. 8   E. $4+4\sqrt{2}$

12. When a water tank is 60% empty, it contains 75 gallons less than when it is 90% full.
    How many gallons of water could the tank hold?
    A. 75   B. 150   C. 225   D. 250   E. 300

13. Two circles, A and B, overlap each other. The area of the overlap is $\dfrac{2}{5}$ the area of circle A and is $\dfrac{5}{8}$ the area of circle B. What is the ratio of the radius of circle A to the radius of circle B?
    A. $\dfrac{1}{4}$ B. $\dfrac{4}{5}$ C. $\dfrac{5}{4}$ D. $\dfrac{4}{3}$ E. $\dfrac{4}{1}$

14. If $a\#b=\dfrac{a+b}{a}$, find $3\#(4\#5)$.
    A. $\dfrac{7}{22}$ B. $\dfrac{4}{7}$ C. $\dfrac{7}{4}$ D. $\dfrac{8}{3}$ E. $\dfrac{22}{7}$

15. In triangle ABC, $AB=4$, $m\angle B=45°$ and $m\angle C=30°$.
    What is the area of the triangle, to the nearest hundredth?
    A. 5.46   B. 8.57   C. 9.24   D. 9.86   E. 10.93

16. Let P be the intersection of the three angle bisectors of a right triangle with side lengths 3, 4, and 5.
    What is the longest distance between P and a point on the triangle, to the nearest hundredth?
    A. 1.41   B. 2.24   C. 3.14   D. 3.16   E. 3.46

17. Jake started adding the consecutive positive integers, $1+2+3+\ldots$, and stopped when the next integer would have put his sum over 1000. How many terms did Jake add to get his sum?
    A. 33   B. 44   C. 55   D. 66   E. 77

18. A chemist has a 1 gallon solution that consists of 1 part ethanol and 2 parts water. The chemist also has a 2 gallon solution that consists of 3 parts ethanol and 5 parts water. If the two solutions are mixed to form a 3 gallon solution, what is the ratio of ethanol to water in the new solution?

    A.   4 to 7   B.   5 to 9   C.   7 to 12   D.   10 to 17   E.   13 to 23

19. Five friends decide to split $200 dollars amongst themselves in the following way. Person #2 will receive twice as much money as Person #1. Person #3 will receive one-seventh as much money as Person #1. Person #4 will receive five times as much money as Person #2. Person #5 will receive $8. Let X be the underlined combined money, in dollars, received by Person #3 and Person #4. Then X satisfies

    A. $0 \le X \le 40$ B. $40.01 \le X \le 80$ C. $80.01 \le X \le 120$ D. $120.01 \le X \le 160$ E. $160.01 \le X \le 200$

20. A swinging pendulum has a time period of 1.83 seconds. A fast video camera is taking pictures of the swinging pendulum every 732 milliseconds. How long should the camera operate to get two identical pictures?

    A. $\dfrac{(0.732)(1.83)}{\text{seconds}}$ B. $\dfrac{(732)(1.83)}{\text{seconds}}$ C. $(2)(3)(5)(61)$ milliseconds   D. $(2)(2)(3)(5)(61)$ milliseconds   E.   Forever

21. If a rectangular region having an area of 500 square units has its length increased by 20 percent and its width decreased by 10 percent, what is the area, in square units, of the newly formed region?

    A.   440   B.   540   C.   550   D.   600   E.   660

22. 15.2% of households in a local town do not own a television, 24.1% own 1 television, 35.6% own 2 televisions, and 25.1% own 3 televisions. Which of the following is the best approximation for the average number of televisions per household in this town?

    A.   1.25   B.   1.4   C.   1.5   D.   1.7   E.   2

23. Five identical marbles are randomly distributed to 3 children. (It is possible that some children receive no marbles.) What is the probability that each child receives at least one marble?

    A. $\dfrac{2}{7}$ B. $\dfrac{2}{5}$ C. $\dfrac{3}{7}$ D. $\dfrac{4}{7}$ E. $\dfrac{3}{5}$

24. If $2x^4 - 3x^3 + ax^2 + 7x + b$ is divisible by $x^2 + x - 2$, then $\dfrac{a}{b}$ is:

    A. -12 B. -6 C. -2 D.   2   E.   6

Use the diagrams below to answer questions 25 and 26.
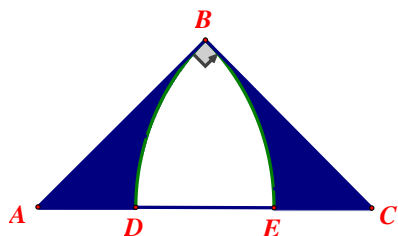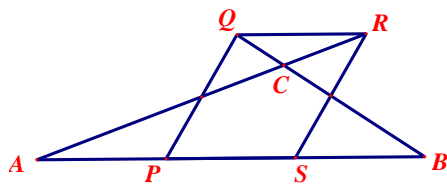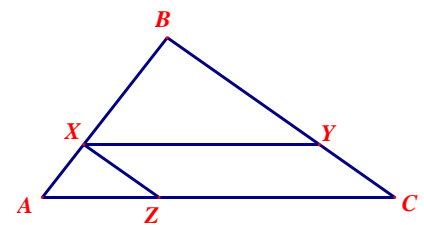

QUESTION 25
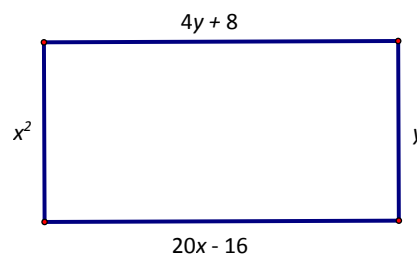
QUESTION 26

25. Triangle ABC is an isosceles, right triangle with $BC = AB = 2$. A circular arc of radius 2 with center C meets the hypotenuse at D. A circular arc of radius 2 with center A meets the hypotenuse at E.
    Which of the following best approximates the area of the shaded region?

    A.   0.43   B.   0.86   C.   1.14   D.   1.5   E.   1.57

26. A rectangle whose sides are presented in terms of x and y is shown in the figure above. Two of the sides have an odd integer length. Find the perimeter of the rectangle.

    A.   16   B.   56   C.   106   D.   156   E.   242

27. Let $f(x)=x^2-3x+2$. For what values of t does $f(t-3)=0$?

    A. 0 and 3   B. -3 and 0   C. 1 and 2   D. -5 and -4 E. 4 and 5

28. Suppose a bag of 3 apples, 7 oranges and 11 pears cost \$6.04. A bag of 2 apples, 5 oranges and 8 pears costs \$4.31. What is the cost of a bag of fruit consisting of 1 apple, 1 orange and 1 pear?

    A. \$0.88   B. \$1.09   C. \$1.73   D. \$2.09   E. \$3.64

29. Flip a fair coin n times.
What is the smallest possible value of n such that the probability of at least two heads exceeds 0.8?

    A. 4   B. 5   C. 6   D. 7   E. 8

30. If nine pens cost eleven dollars and x cents, and thirteen pens cost fifteen dollars and y cents, then $x+y$ is:

    A. 70   B. 84   C. 106   D. 107   E. 112

31. The number of distinct positive integral divisors of 30^4, excluding 1 and 30^4, is:

    A. 30   B. 110   C. 123   D. 125   E. 243

32. A large circular piece of plywood has an area of 9π square feet. A carpenter plans to cut four congruent, maximum-sized circular pieces from the plywood.
What is the radius, in feet, of each of the four congruent circular pieces?

    A. $-3+\sqrt{2}$ B. $-3+3\sqrt{2}$ C. $3-\sqrt{2}$ D. $3+\sqrt{2}$ E. $3+3\sqrt{2}$

33. Ten people play a round-robin chess tournament (every person plays everyone else exactly once). In each game, the winner gets 3 points, the loser gets 0 points, and in the case of a draw (tie), each of the players gets 1 point. The total number of points awarded in the tournament is 131. How many games are draws?

    A. 1   B. 2   C. 3   D. 4   E. 5

34. Find the number of solutions to the equation $\left|-x\right|=-x$.

    A. zero   B. one   C. two   D. three   E. more than three

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Use the diagrams below to answer questions 35 and 36.

QUESTION 35   QUESTION 36

35. In the diagram above, $\overline{AR}$ bisects $\overline{PQ}$, $\overline{BQ}$ bisects $\overline{SR}$, and the area of parallelogram $PQRS$ is k.
What is the area of triangle ABC?

    A. k B. $\dfrac{9k}{8}$ C. $\dfrac{6k}{5}$ D. $\dfrac{5k}{4}$ E. $2k$

36. In the diagram, $\overline{XY}$ is parallel to $\overline{AC}$, and $\overline{XZ}$ is parallel to $\overline{BC}$. Triangle ABC has an area of 98 square units, and trapezoid $XYCA$ has an area of 48 square units. What is the area, in square units, of trapezoid $BCZX$?

    A. 50   B. 56   C. 70   D. 76   E. 90