2017 SCSU MATH CONTEST
9th and 10th Grade Test – 50th Annual Edition

DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

1. This is the 50th annual SCSU Math Contest. In honor of this, find the ones digit of 2017^{50}.
   A. 0 B. 1 C. 3 D. 7 E. 9

2. At Perry's Parers, each parer pares a pair of pears every 6 minutes. How many pears do a pair of triplets pare in a pair of hours?
   A. 60 B. 120 C. 160 D. 240 E. 480

3. Assume $x>0$ and $y>0$. Then $\sqrt{x}+\sqrt{y}-\sqrt{x+y}$ is:
   A. Always positive B. Sometimes zero C. Sometimes negative D. Sometimes Undefined E. Impossible to determine

4. Consider the page numbers on eight consecutive pages of a book.
   The sum of these eight page numbers could **NOT** be:
   A. 612 B. 780 C. 864 D. 932 E. 1148

5. Jake has only nickels, dimes, and quarters in his pocket. The value of the dimes is the same as the value of the quarters, and he has 7 fewer nickels than dimes. The value of these coins is $5.90. How many coins are in Jake's pocket?
   A. 18 B. 25 C. 28 D. 31 E. 53

6. The first SCSU Math Contest was held in 1968. In honor of this, find the sum of the **odd** integers from 1968 through 2017 (including 2017).
   A. 47,818 B. 49,800 C. 49,825 D. 51,792 E. 51,817

7. Three automated machines are used to make a product. The two newer machines are each faster than the old machine. Each new machine produces two less than twice the number of units per hour that the old machine produces. All three machines work 7.5 hours per day for 5 days and produce 1725 units. How many units per hour does the old machine produce?
   A. 10 B. 12 C. 15 D. 18 E. Impossible to determine

8. Let $f(n)=n\div\left[(n-1)\div\left[(n-2)\div\left[\cdots(3\div(2\div1))\right]\right]\right]$ for $n=2,3,4,\ldots$. Which of the following is true?
   A. $f(7)=1$ B. $f(7)=\frac{7\cdot5\cdot3}{6\cdot4\cdot2}$ C. $f(7)=\frac{6\cdot4\cdot2}{7\cdot5\cdot3}$ D. $f(7)=7\cdot6\cdot5\cdot4\cdot3\cdot2$ E. None of these

9. Millie was born 1 million seconds after the start of 2017. If New Year's Day 2017 was Sunday, on what day of the week was Millie born?
   A. Wednesday B. Thursday C. Friday D. Saturday E. Sunday

10. Let a and b be positive real numbers with $a+b=4$. What is the minimum value of $\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)$?

A. 2 B. $\frac{8}{3}$ C. $\frac{9}{4}$ D. 3 E. 4

11. The lengths of the sides of a right triangle are all integers. Two of these integers are primes that differ by 50. Compute the smallest possible value for the length of the third side.

A. 51 B. 53 C. 56 D. 60 E. 61

12. Four suspects of a crime made the following statements to the police.

Anil: Cora stole the $50.
Berta: I did not steal the $50.
Cora: Dan stole the $50.
Dan: Cora lied when she said that I stole the $50.

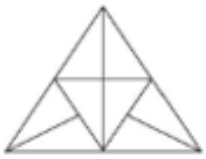
Given that one of them "did it" and that exactly one of them told the truth, who stole the $50?

A. Anil B. Berta C. Cora D. Dan E. Impossible to determine

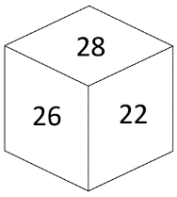
13. To celebrate the 50th annual SCSU Math Contest, Jen wants to walk 50% farther today than yesterday in only 50% of the time she walked yesterday. What percent <u>increase</u> in speed will she need today as compared to yesterday to meet her goal?

A. 20% B. 50% C. 200% D. 300% E. 500%

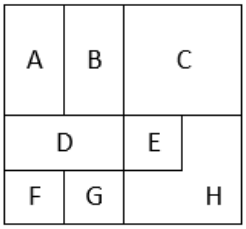
Use the three figures below to answer questions 14, 15, and 16.



Question 14



Question 15



Question 16

14. How many triangles of any size are in the picture, above left?

A. 12 B. 13 C. 15 D. 17 E. 18

15. The six numbers on the faces of our cube, above center, are consecutive even numbers. The sums of the two numbers on each pair of opposite faces are equal. A portion of our cube can be seen in the diagram. What is the number on the face opposite 22?

A. 18 B. 20 C. 24 D. 30 E. 32

16. Eight identical sheets of paper were placed on the table one at a time, overlapping as shown in the diagram, above right. Which sheet(s) of paper could have been the fifth one placed on the table?

A. Only A B. Only D C. Only E D. Only D or E E. Only E or H

17. For what positive real number n are $\frac{6+4\sqrt{n}}{n}$ and $\frac{6-4\sqrt{n}}{n}$ reciprocals?

A. 2 B. $4+4\sqrt{3}$ C. 12 D. 16 E. 18

18. Let x be a solution of $4^x - 4^{x-1} = 24$. Find $(2x)^x$.

A. $\sqrt{5}$ B. $5\sqrt{5}$ C. 25 D. $25\sqrt{5}$ E. 125

19. The perimeter of a rhombus is 60 units. One diagonal is 24 units long. Find the length of the other diagonal.

A. 9 units B. 12 units C. 18 units D. 24 units E. None of these

20. Two fair dice are rolled. A player gets two points if both faces show different prime numbers, and zero points otherwise. A player rolls these two dice twice. What is the probability that she has a score of two points at the end of these two rolls?

A. $\frac{5}{18}$ B. $\frac{5}{36}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. $\frac{1}{9}$

21. How many different ways are there to get to the top of a nine-step stairway if you take either one step or two steps at a time?

A. 34 B. 54 C. 89 D. 144 E. None of these

22. The sum of the digits of 666^2 is 27.
Find the sum of the digits of $666,666,666,666^2$.

A. 72 B. 84 C. 86 D. 108 E. 120

23. Find the point on the y-axis that is equidistant from the points $(2,1)$ and $(4,-3)$.

A. $\left(0,-\frac{5}{2}\right)$ B. $(0,-2)$ C. $(-2,0)$ D. $(3,-1)$ E. $(0,-1)$

24. If $xy = a$ and $\frac{1}{x^2}+\frac{1}{y^2} = b$, then find $(x+y)^2$.

A. $(2a+b)^2$ B. a^2+b^2 C. $a(ab+2)$ D. $ab(b+2)$ E. $2a+\frac{1}{b}$

25. Consider a binary operation * defined by $a*b = a^b$. For all positive integers a, b, and c, which of the following is true?

A. $a*b^c = (a*c)*b$ B. $a*(b*c) = (a*b)*c$ C. $a*b = b*a$ D. $(a*b)^c = a*(bc)$ E. None of these

26. An equilateral triangle and a regular hexagon have equal perimeters. The area of the triangle is 2 square units. Find the area of the hexagon in square units.

A. 3 B. 4 C. 6 D. 8 E. 12

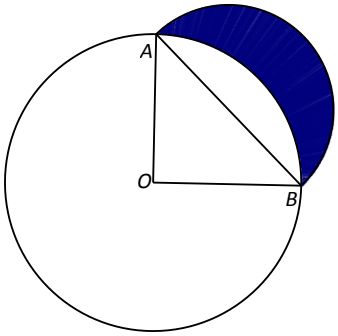
27. Let $x>0$ and $b>0$. Which of the following is an equivalent expression for $\log_b 2+\log_b x-\log_b(x+3)$?

A. $\log_b 2-\log_b 3$ B. $\log_b\left(\frac{2x}{x+3}\right)$ C. $\log_b\left(\frac{3}{2}\right)$ D. $\log_b\left(\frac{2+x}{3x}\right)$ E. $\frac{\log_b(2x)}{\log_b(x+3)}$

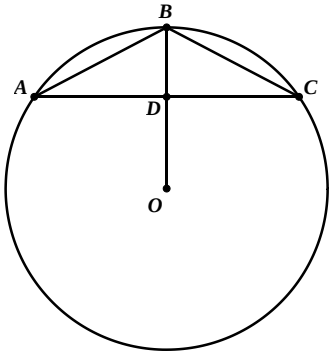
Use the three figures below to answer questions 28, 29 and 30.

1
3 5
7 9 11
13 15 17 19
21 23 25 27 29
31 33 35 37 39 41

Question 28



Question 29



Question 30

28. The first six rows of a triangular array of odd numbers is shown, above left.
What will be the sum of the first number and the last number in row 17?

A. 572 B. 578 C. 582 D. 594 E. 598

29. In a circle with center *O*, *OA* and *OB* are radii and $\angle$AOB is a right angle, as shown above center. A semicircle is constructed using segment AB as its diameter as shown. The shaded portion of the semicircle outside circle *O* is called a *lune*. What is the ratio of the area of the *lune* to the area of the triangle?

A. $\sqrt{2}:\pi$ B. $1:1$ C. $\frac{\pi}{\sqrt{3}}:1$ D. $\pi:\sqrt{2}$ E. $\frac{\pi}{3}:1$

30. In the figure shown, above right, the radius of the circle is $OC=1$ and $\overline{OB}\perp\overline{AC}$. Express the area of ΔABC as a function of *x*, where $OD=x$.

A. $A(x)=(1-x)\sqrt{1-x^2}$ B. $A(x)=\frac{1}{2}x(1-x)$ C. $A(x)=\frac{1}{2}\sqrt{1-x^2}$ D. $A(x)=\frac{1}{2}$ E. None of these

31. Let $f(x)=x^2+x-6$. For what values of t does $f(t-5)=0$?

A. -3 and 2 B. -2 and 3 C. 5 D. 2 and 7 E. None of these

32. Find the remainder when $x^{50}+50$ is divided by $x+1$.

A. 0 B. 1 C. 49 D. 50 E. 51

33. In a triangle ΔABC, $AC=6$ and $BC=7$. Suppose the median from vertex *A* is perpendicular to the median from vertex *B*. Find *AB*.

A. 4 B. $\sqrt{17}$ C. $2\sqrt{5}$ D. 5 E. $\sqrt{85}$

34. Let $A(-1,-2)$ and $B(7,14)$ be points in the plane. Find an equation of the perpendicular bisector of line segment $\overline{AB}$.

A. $2x+y=15$ B. $x+2y=15$ C. $-2x+y=0$ D. $-x+2y=3$ E. $2x-y=3$