2016 SCSU MATH CONTEST
9^{th} and 10^{th} GRADE

DIRECTIONS: Select the <u>BEST</u> completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

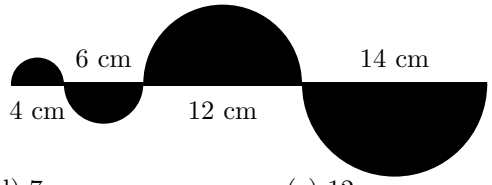
1. Find the sum of all the odd whole numbers that are less than 2016.

   (a) 508,536 (b) 1,016,064 (c) 1,016,568 (d) 2,032,128 (e) 2,033,136

2. The *prime factorization* of 600 is $2^3 \times 3 \times 5^2$, so 600 is the product of powers of three distinct prime factors (2, 3, and 5). How many distinct prime factors does the number 2016 have?

   (a) 2 (b) 3 (c) 4 (d) 5 (e) 6

3. Semicircles are formed on alternating sides of a 36 cm segment, as indicated. In centimeters, what is the radius of a circle that has the same area as the shaded region?



   (a) $3\sqrt{2}$ (b) 4.5 (c) 6 (d) 7 (e) 12

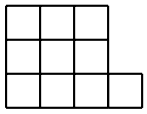
4. The distance between the centers of two consecutive posts in a straight fence along a property line is 3.5 meters. The distance from the center of the fourth post to that of the middle post is 112 meters. How many posts are in the fence?

   (a) 35 (b) 36 (c) 70 (d) 71 (e) 72

5. On a 25-question test, a student scores four points for each correct answer and loses two points for each incorrect answer. Sandi answered every question on the test and obtained a score of 64 points. How many questions did Sandi answer correctly?

   (a) 6 (b) 17 (c) 18 (d) 19 (e) none of these

6. How many different rectangles are in the figure shown to the right?



   (a) 31 (b) 33 (c) 34 (d) 39 (e) 40

7. An open top cubical box is completely filled with 512 unit cubes. How many of the unit cubes do not touch either a side or the bottom of the box?

   (a) 216 (b) 252 (c) 412 (d) 448 (e) 496

8. The faces of a fair 8-sided die are numbered with the numerals 1-8, one numeral per face. This die is rolled twice. Find the probability that the sum of the numbers on the top face is a prime number.

   (a) $\frac{5}{64}$ (b) $\frac{3}{32}$ (c) $\frac{14}{64}$ (d) $\frac{23}{64}$ (e) $\frac{41}{64}$

9. The average age of a class of 30 students is 29 years. The average age of the boys is 32 years and the average age of the girls is 23 years. How many girls are in the class?

   (a) 10 (b) 15 (c) 20 (d) 25 (e) None of these

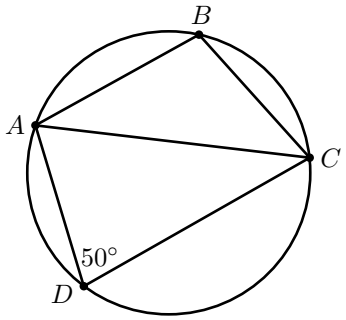
10. If $a_1 = 3$, $a_2 = 5$, and $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$, determine the value of the 2016^{th} term.

    (a) -5 (b) -2 (c) 2 (d) 3 (e) 5

11. The average price of a three-bedroom house in Saint Cloud in 2016 is \$140,000. Prices have depreciated 3% each year for each of the last five years. To the nearest dollar, determine the average price of a three-bedroom house in Saint Cloud in 2013 (three years ago).

    (a) \$147,793 (b) \$149,793 (c) \$152,600 (d) \$152,982 (e) \$153,396

12. Given: Segments AB and CD are parallel (see figure, not to scale, at the right).
$m\angle ADC = 50°$.
$m\angle BAC = m\angle BCA$.
Find $m\angle BAD$.



(a) 90° (b) 100° (c) 110° (d) 120° (e) 130°

13. The average number of hot dogs that three boys ate was 16. The average number of hot dogs that two girls ate was 6. What is the average number of hot dogs eaten per person?

(a) 11 (b) 12 (c) 13 (d) 14.5 (e) 18

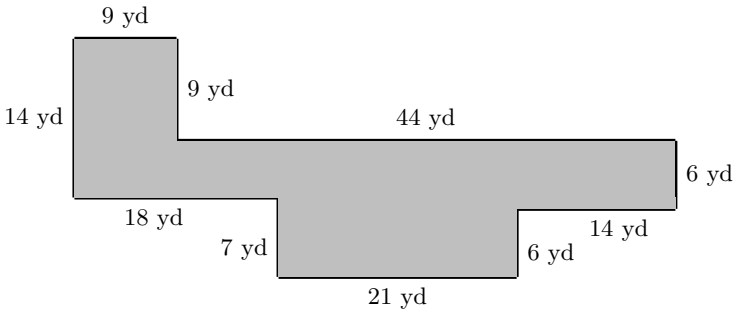
14. A bakery makes its popular swirl brownie mix by mixing two kinds of batter. The peanut butter batter costs \$2.15 per liter, and the chocolate batter costs \$1.49 per liter. When they are mixed together to create the swirl batter, it ends up costing \$1.85 per liter. If the bakery already has 30 liters of peanut butter batter, how many liters of chocolate batter will be needed to make the swirl batter?

(a) 19 (b) 22 (c) 25 (d) 28 (e) 31

15. In a swimming relay race, Amos swims the first lap in 36 seconds. Bob swims the second lap, slightly slower, at 90% of Amos' *speed*. Curt swims the third lap at $\frac{5}{4}$ Bob's speed. Dirk anchors the race by swimming the final lap in the average of Amos', Bob's, and Curt's times. What was the team's final relay time?

(a) 2:05.8 (b) 2:24.0 (c) 2:25.2 (d) 2:31.2 (e) 2:48.0

16. Determine the area of the given shape.



(a) 493 sq. yd. (b) 507 sq. yd. (c) 520 sq. yd. (d) 537 sq. yd. (e) 549 sq. yd.

17. Two positive real numbers are reciprocals that differ by two. Which one of the following represents their sum?

(a) $\sqrt{2}$ (b) $1+\sqrt{2}$ (c) $1+\sqrt{3}$ (d) $2\sqrt{2}$ (e) $2\sqrt{3}$

18. Dad baked a cake for the entire family. Mom ate one-sixth of the cake; the son ate one-fifth of the remaining cake; the daughter ate one-fourth of what was left after that; the dog ate one-third of what was left after that and the baby ate one-half of what was then remaining. How much of the original cake was left for Dad to eat?

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$ (e) $\frac{1}{2}$

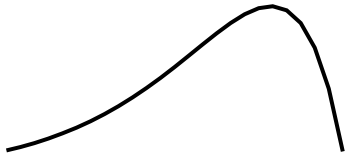
19. A race has several staggered starting times - 9:30 a.m., 10:15 a.m., 11:30 a.m., 11:45 a.m., and 1:30 p.m.. What is the average start time?

(a) 10:16 a.m. (b) 11:00 a.m. (c) 11:10 a.m. (d) 11:18 a.m. (e) 11:30 a.m.

20. Consider a list of numbers $10^{1/13}, 10^{2/13}, 10^{3/13}, \ldots, 10^{n/13}$. Find the least positive integer n such that the product of the first n terms exceeds 100,000.

(a) 8 (b) 9 (c) 10 (d) 11 (e) 12

21. When data is skewed left (negatively skewed), and M denotes the median, the mean will usually be



(a) greater than M. (b) negative. (c) equal to M. (d) positive. (e) smaller than M.

22. If $3^{2x} + 9 = 10(3^x)$, determine the value of $x^2 + 1$.

(a) 1 only (b) 2 only (c) 5 only (d) 1 or 2 (e) 1 or 5

23. Determine values of k such that the function $f(x) = kx^2 + x + k$ has a repeated real zero.

(a) $k = \pm\frac{1}{2}$ (b) $k = \pm 2$ (c) $k = \pm\frac{2}{3}$ (d) $k = \pm\frac{1}{3}$ (e) There are no such values of k.

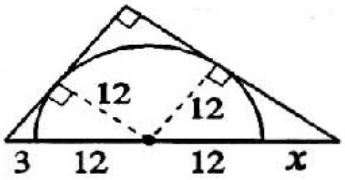
24. Suppose $N = 1 + 11 + 101 + 1001 + \ldots + 1\overbrace{000\cdots000}^{\text{2016 zeroes}}1$. When N is written as a single integer, determine the sum of its digits.

(a) 11 (b) 2017 (c) 2021 (d) 4031 (e) 4035

25. Andrea took her first dose, 2016 mg of medication, at 2:00 p.m.; another 2016 mg at 6:00 p.m.; and her final dose of 2016 mg at 10:00 p.m. However, 80% of the medication is cleared from her body every two hours. Rounded to the nearest milligram, how many milligrams of medication remain in Andrea's body at midnight?

(a) 216 (b) 404 (c) 420 (d) 500 (e) 3306

26. A semicircle is tangent to both legs of a right triangle and has its center on the hypotenuse. The hypotenuse is portioned into four segments with measures 3, 12, 12 and x as shown. Find the value of x.



(a) 2 (b) 3 (c) 4 (d) 5 (e) 8

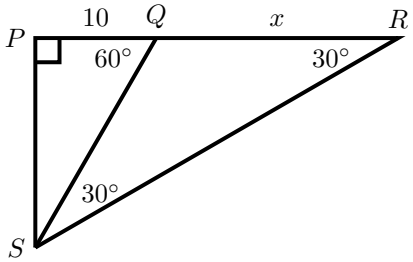
27. Determine the values of x for which the equality $\log(x+3) + \log(x-1) = \log(x^2 - 2x - 3)$ is satisfied.

(a) all real values of x (b) no real values of x (c) all real values of x except $x = 0$ (d) no real values of x except $x = 0$ (e) all real values of x except $x = 1$

28. A painting $18'' \times 24''$ is to be placed into a wooden frame with the longer dimension vertical. The wood at the top and the bottom is twice as wide as the wood on the sides. If the frame area equals that of the painting itself, determine the ratio of the smaller dimension to the larger dimension of the outside of the wooden frame.

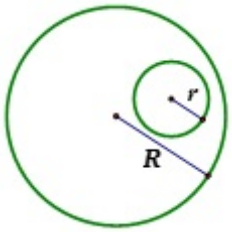
(a) 1 : 3 (b) 1 : 2 (c) 2 : 3 (d) 3 : 4 (e) 1 : 1

29. In the diagram, $PQ = 10$ and $QR = x$. Determine the value of x.



(a) $10\sqrt{3}$ (b) 20 (c) $\frac{50}{3}$ (d) $\frac{20}{\sqrt{3}}$ (e) 10

30. A circle with radius r is contained within the region bounded by a circle with radius R. The ratio of the area bounded by the large circle and the area of the region outside the smaller circle and inside the large circle is $x : y$. Which of the following represents the ratio $R : r$?



(a) $\sqrt{x} : \sqrt{y}$ (b) $x : \sqrt{x-y}$ (c) $y : \sqrt{x-y}$ (d) $\sqrt{x} : \sqrt{x-y}$ (e) $\sqrt{y} : \sqrt{x-y}$

31. Bob the builder was plastering a wall while standing on a ladder. He noticed that the number of rungs below the rung he was standing on was one-third the number of rungs above where he was standing. He then climbed another ten rungs and noticed that the number of rungs below and above where he was standing was then equal. How many rungs are on his ladder?

(a) 30 (b) 35 (c) 40 (d) 41 (e) 61

32. Penny emptied her coin purse. She had 27 coins - pennies, nickels, dimes, and quarters - totaling \$3.30. If the pennies were quarters and the quarters were pennies, she would have only \$2.34. How much money would she have if the nickels were dimes and the dimes were nickels (pennies remain pennies, and quarters remain quarters)?

(a) \$2.65 (b) \$2.82 (c) \$3.25 (d) \$3.50 (e) \$3.80

33. A *stairstep number* is a number (not starting with zero) whose digits are strictly increasing in value from left to right. How many seven-digit stairstep numbers exist?

(a) 12 (b) 36 (c) 72 (d) 120 (e) 720

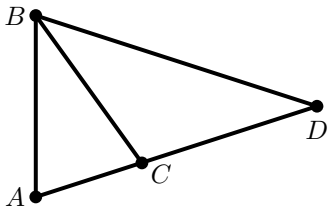
34. Let $f(x) = |x-2| + |x-4| - |2x-6|$ for $2 \leq x \leq 8$. Determine the sum of the largest and smallest values of $f(x)$.

(a) 1 (b) 2 (c) 4 (d) 6 (e) 7

35. January 1st, 2016 was a Friday. Carl went to the gym. He volunteered at the local food bank the next day. Carl plans to keep a resolution in which he goes to the gym every other day and volunteers at the food bank each Saturday. How many times in 2016 will Carl both volunteer and go to the gym? [Note: 2016 is a leap year.]

(a) 26 (b) 27 (c) 28 (d) 51 (e) 52

36. In the given diagram, $AB = BC = CD$ and $AD = BD$. What is the measure of angle D?



(a) 15° (b) 20° (c) 27° (d) 30° (e) 36°