## 2013 SCSU MATH CONTEST 9<sup>th</sup> and 10<sup>th</sup> Grade Test

**DIRECTIONS:** Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

| 1.  | 1. When I saw Tom, he was part way up a ladder. He went up four rungs, down seven rungs and up ten. That put<br>Tom at the top rung of the ladder (don't try that at home!). Then he went down nine rungs, up three rungs<br>and down 10. That put him at the bottom of the ladder, that is, with his feet on the ground. How many rungs<br>are on Tom's ladder? |    |                   |    |                 |    |                  |    |                 |
|-----|--|----|-------------------|----|-----------------|----|------------------|----|-----------------|
| a.  | 15   | b. | 16                | c. | 17              | d. | 18               | e. | 19              |
| 2.  | Write $\sqrt[3]{m^2} \cdot \sqrt[5]{m^3}$ as a single radical.   |    |                   |    |                 |    |                  |    |                 |
| a.  | $m^{15}\sqrt{m^4}$   | b. | $m^2 \sqrt[5]{m}$ | c. | $\sqrt[8]{m^5}$ | d. | $m\sqrt[5]{m^2}$ | e. | $\sqrt[5]{m^2}$ |
| 3.  | Jake spent half of his money, then lost one-third of what remained. He then spent \$4, leaving him with \$18.<br>How much money did Jake have to begin with?   |    |                   |    |                 |    |                  |    |                 |
| a.  | \$60   | b. | \$66              | c. | \$72            | d. | \$88             | e. | \$132           |
| 4.  | Mandi bought a DVD player that was on sale for 20% off the regular selling price. After the 5% sales tax was added, she paid \$83.58 for the player. What was the regular selling price of the DVD player?   |    |                   |    |                 |    |                  |    |                 |
| a.  | \$96.12  | b. | \$99.50           | c. | \$100.30        | d. | \$105.30         | e. | \$105.57        |
| 5.  | Jerry buys roses at 3 for \$5 and sells them at 4 for \$10.00. How many roses will Jerry have to buy and sell in<br>order to make a \$50.00 profit?  |    |                   |    |                 |    |                  |    |                 |
| a.  | 24   | b. | 36                | c. | 48              | d. | 60               | e. | 72              |
| 6.  | A set of nine consecutive odd whole numbers has a sum of 1359. What is the sum of the smallest and largest of these nine numbers?  |    |                   |    |                 |    |                  |    |                 |
| a.  | 248  | b. | 270               | c. | 276             | d. | 284              | e. | 302             |
| 7.  | The number, V, of computers infected by a computer virus is modeled by $V = 10e^{4.6052t}$ , where t is the time in hours. Which is the best estimate of the number of computers that will be infected after 2 hours?  |    |                   |    |                 |    |                  |    |                 |
| a.  | 250  | b. | 1995              | c. | 2041            | d. | 20,001           | e. | 100,006         |
| 8.  | A bank has been charging 25 cents a month plus 10 cents a check for a checking account, but is changing the price to 50 cents a month plus 8 cents per check written. What is the smallest number of checks that must be written in one month for the new plan to save a client money for that month?  |    |                   |    |                 |    |                  |    |                 |
| a.  | 10   | b. | 11                | c. | 12              | d. | 13               | e. | 15              |
| 9.  | A manager ordered netbooks for the people in her department. Including tax, the top line model each cost<br>\$500 and the cheaper model each cost \$250. She ordered three times as many top line netbooks as cheaper<br>ones. The total cost for the order was \$7000. How many top line netbooks did she order?  |    |                   |    |                 |    |                  |    |                 |
| a.  | 4  | b. | 7                 | c. | 9               | d. | 12               | e. | 28              |
| 10. | What is the maximum number of points of intersection when two circles and five straight lines intersect?   |    |                   |    |                 |    |                  |    |                 |

a. 10 b. 20 c. 26 d. 30 e. 32

- 11. A math test has 20 questions. Each correct answer is 3 points and every wrong answer deducts 1 point. Qiang earned a score of 44 and didn't leave any questions blank. How many questions did Qiang get correct?
- a. 8 b. 12 c. 13 d. 16 e. 18

12. Crane R can unload a truck in 5 hours. Crane G can unload the same amount in 4 hours. Crane R started unloading a truck at 10:00 am. Crane G became free and started helping unload the truck at noon. At what time was the unloading complete?

- a. 12:27 p.m. b. 1:00 p.m. c. 1:20 p.m. d. 2:00 p.m. e. 2:23 p.m.
- 13. Factor and simplify  $\frac{a^3-b^3}{a^2-b^2}$
- a.  $\frac{a^2 + ab + b^2}{a + b}$  b. a + b c.  $\frac{a^2 ab + b^2}{a b}$  d. a b e.  $\frac{a^2 + ab b^2}{a + b}$
- 14. If 6 boys fill 6 notebooks in 6 weeks and 4 girls fill 4 notebooks in 4 weeks, how many notebooks will a class of 12 boys and 12 girls fill in 12 weeks?
- a. 20 b. 24 c. 30 d. 36 e. 60

15. A conveyor belt moves at 0.25 feet per second. A machine removes the items at a rate of 6 per minute. How far apart must the items be placed so that the next item will arrive just as the machine is ready to remove it?

a. 18 inches b. 25 inches c. 30 inches d. 36 inches e. 60 inches

16. It's between 11:00 a.m. and noon. In 13 minutes it will be as many minutes short of 1 p.m. as it was past 11 a.m. just seven minutes ago. What time is it now?

- a. 11:34 a.m. b. 11:40 a.m. c. 11:50 a.m. d. 11:53 a.m. e. 11:57 a.m.
- 17. Find the exact value of x if  $14e^{3x+2} = 560$ .
- a.  $\frac{\ln 40 2}{3}$  b.  $\frac{\ln 28}{3}$  c.  $\frac{\ln 16}{3}$  d.  $\frac{40 \ln 2}{3}$  e.  $\frac{\ln 20 4}{3}$

18. In  $\triangle ABC$ , D is on  $\overline{AB}$  and E is on  $\overline{AC}$  such that  $\overline{DE}$  is parallel to  $\overline{BC}$ . If AD = 9, BD = 6, and DE = 4, then BC =

a.  $2\frac{2}{3}$  b.  $5\frac{1}{3}$  c.  $6\frac{2}{3}$  d.  $7\frac{1}{3}$  e.  $13\frac{1}{2}$ 

19. A whale weighs 65,536 pounds on January 1. During January, its weight increases by 50%. Because of a long migration during February, its weight decreases by 50% during that month. But resting happily at its destination during March, its weight increases by 50% during that month. During April it does not have access to enough fish so its weight decreases by 50%. It continues alternately gaining 50% and losing 50% of its weight for the remaining eight months of the year. How many pounds does the whale weigh on Dec. 31?

- a. 11,664 b. 23,328 c. 32,768 d. 65,536 e. 131,075
- 20. Two angles of a quadrilateral have the same measure. The measure of the third angle is equal to the sum of the two equal angles. The measure of the fourth angle is 60° less than twice the sum of the other three angles. Find the measure of the largest angle.

a. 160° b. 180° c. 220° d. 240° e. 300°

| 21.  | If $6x + 7y = 2004$ and $7x + 6y$   | y = 4002, then $x + y =$           |                  |                                       |  |  |  |  |
|--|---|------------------------------------|------------------|---------------------------------------|--|--|--|--|
| a.   | 221 b. 400  | c. 462                             | d. 487           | e. 770                                |  |  |  |  |
| 22.  | Four fair coins are tossed. What is the probability there will be two heads and two tails showing?  |                                    |                  |                                       |  |  |  |  |
| a.   | $\frac{1}{8}$ b. $\frac{1}{4}$  | c. $\frac{3}{8}$                   | d. $\frac{1}{2}$ | e. <u>5</u><br>8                      |  |  |  |  |
|  |   | 0                                  | 2                | 0                                     |  |  |  |  |
| 23.  | Solve $\frac{x^2+3x-4}{x-3} \ge 0$  |                                    |                  |                                       |  |  |  |  |
| a.   | $(-1 \le x < 3) \text{ or } (x \ge 4)$  | b. $(-4 \le x \le 1)$ or $(x > 1)$ | • 3) c.          | $(x \le 1) \text{ or } (3 < x \le 4)$ |  |  |  |  |
| d.   | $(1 \le x < 3) \text{ or } (x \ge 4)$   | e. $(-3 < x \le 1)$ or $(x \ge 1)$ | 4)               |                                       |  |  |  |  |
| 24.  | Each of the "golden arches" at a McDonald's restaurant is in the shape of a parabola. Each arch is modeled<br>by: $h(x) = -x^2 + 6x$ , where $h(x)$ is the height of the arch (in feet) at a distance x (in feet) from one side.<br>How high, in feet, is the arch at its axis of symmetry?   |                                    |                  |                                       |  |  |  |  |
| a.   | 6 b. 9  | c. 12                              | d. 15            | e. 18                                 |  |  |  |  |
| The three remaining questions on this page refer to the figures at the bottom of the page. |   |                                    |                  |                                       |  |  |  |  |
| 25.  | Find the area, in square units, o   |                                    |                  |                                       |  |  |  |  |
| a.   | 8.5 b. 10   | c. 11                              | d. 11.5          | e. 13                                 |  |  |  |  |
| 26.  | In Figure 2 (below, center), ABC  | DE is a regular pentagram. Fin     | d the measure of | angle A.                              |  |  |  |  |
| a.   | 18° b. 24°  | c. 30°                             | d. 36°           | e. 40°                                |  |  |  |  |
| 27.  | A sign is held up with a brace as shown in Figure 3 (below, right). The distance from the bottom of the brace to the bottom of the sign is 2 feet. To be effective, the top of the brace must reach at least $\frac{2}{3}$ the height of the sign. What is the maximum height the sign can be if the length of the brace is 8 feet? |                                    |                  |                                       |  |  |  |  |
| a.   | 7.75 feet b. 11.62 fee  | et c. 12.37 feet                   | d. 26.67 feet    | e. 34.86 feet                         |  |  |  |  |
| 1 u  |   |                                    | D                | Sign<br>brace<br>7<br>X 2 feet x      |  |  |  |  |
|  | Figure 1  | Figure 2                           |                  | Figure 3                              |  |  |  |  |

| 28. | What is the units  | digit of 3 <sup>2013</sup> ? |                            |                            |                            |  |  |
|-----|--|------------------------------|----------------------------|----------------------------|----------------------------|--|--|
| a.  | 1  | b. 3                         | c. 6                       | d. 7                       | e. 9                       |  |  |
| 29. | How many natural number pairs of $(x, y)$ satisfy the equation $20x + 13y = 2013$ ?  |                              |                            |                            |                            |  |  |
| a.  | 5  | b. 6                         | c. 7                       | d. 8                       | e. 9                       |  |  |
| 30. | Find the standard form of the equation of the circle for which the endpoints of a diameter are $(-2, -2)$ and $(4, -6)$ .  |                              |                            |                            |                            |  |  |
| a.  | $(x+1)^2 + (y-x)^2$  | $(4)^2 = 13$ b.              | $(x-1)^2 + (y+4)^2 = 13$   | B c. $(x-4)$               | $y^2 + (y+6)^2 = 52$       |  |  |
| d.  | $(x+1)^2 + (y-x)^2$  | $(4)^2 = 52$ e.              | $(x-2)^2 + (y-2)^2 = 13$   | 3                          |                            |  |  |
| 31. | Find the equation of the line that passes through $(2, -1)$ and is perpendicular to $2x - 3y = 5$ .  |                              |                            |                            |                            |  |  |
| а.  | $y = \frac{-3}{2}x + 3$  | b. $y = \frac{-2}{3}x + 5$   | c. $y = \frac{-2}{3}x + 2$ | d. $y = \frac{-3}{2}x + 5$ | e. $y = \frac{-3}{2}x + 2$ |  |  |
| 32. | second die will be larger than the number on the first die?  |                              |                            |                            |                            |  |  |
| a.  | 5<br>36  | b. $\frac{5}{12}$            | c. $\frac{11}{36}$         | d. $\frac{1}{2}$           | e. $\frac{5}{6}$           |  |  |
| 33. | A cylinder holds five spheres of diameter <i>D</i> , in one row, with no extra room on the top, bottom or sides. Which volume below, in cubic units, most closely approximates the volume of this cylinder?                  |                              |                            |                            |                            |  |  |
| a.  | $\frac{5}{4}\pi D^3$   | b. $\frac{5}{6}\pi D^3$      | c. $\frac{20}{3}\pi D^3$   | d. $5\pi D^3$              | e. $20\pi D^3$             |  |  |
| 34. | . Two fair dice are rolled, and the difference between the two numbers on the dice is seen to be 4. What is the probability that a third die rolled will be no smaller than the larger of the first two dice?                |                              |                            |                            |                            |  |  |
| a.  | $\frac{1}{4}$  | b. $\frac{1}{12}$            | c. $\frac{7}{36}$          | d. $\frac{1}{108}$         | e. $\frac{5}{108}$         |  |  |
| 35. | Which one of the following functions meets these two criteria:<br>f(0) = 0 and, for all $x$ , $f(x + 1) = 2f(x) + 1$ ?   |                              |                            |                            |                            |  |  |
| a.  | $f(x) = 1 - 2^x$   | b. $f(x) = 2x$               | c. $f(x) = 2x^2 - x$       | d. $f(x) = -2^x - 1$       | e. $f(x) = 2^x - 1$        |  |  |
| 36. | . A farmer with 2000 feet of fencing wants to enclose a rectangular plot that borders on a straight river. If the farmer does not fence the side along the river, what is the largest rectangular area that can be enclosed? |                              |                            |                            |                            |  |  |
| a.  | 95,000 ft <sup>2</sup>   | b. 320,000 ft <sup>2</sup>   | c. 500,000 ft <sup>2</sup> | d. 600,000 ft <sup>2</sup> | e. 640,000 ft <sup>2</sup> |  |  |
| 37. | 7. A student dance committee, consisting of 2 boys and 3 girls, is to be chosen from among 4 boys and 8 girls.<br>How many different committees are possible?  |                              |                            |                            |                            |  |  |
| a.  | 38   | b. 192                       | c. 214                     | d. 336                     | e. 362                     |  |  |

End of exam