2012 SCSU MATH CONTEST
9th and 10th Grade Test

DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

1. A tire on a truck rotates at 150 revolutions per minute when the truck is travelling 40 km per hour. What is the best estimate for the circumference of this tire?

   a. 2 meters   b. 3.75 meters   c. 4 meters   d. 4.44 meters   e. 5 meters

2. Let $S_n = 1 - 2 + 3 - 4 + \cdots + (-1)^{n-1} n$. Then $S_{47} + S_{53} + S_{100}$ equals

   a. −2   b. −1   c. 0   d. 1   e. 2

3. The equation of the line perpendicular to y = 5 and containing the point (5, -5) is

   a. $y = -5$ b. $x + y = 5$ c. $y = x$ d. $x + y = 0$ e. $x = 5$

4. A car drives the first 60 miles at 60 miles per hour and the second 60 miles at 30 miles per hour. What is the average speed of the car?

   a. 40 miles per hour   b. 42 miles per hour   c. 45 miles per hour
   d. 48 miles per hour   e. 50 miles per hour

5. Simplify: $\dfrac{2}{x+3} - \dfrac{6x}{2x+1}$

   a. $\dfrac{-6x+2}{-x+2}$ b. $\dfrac{-6x^2-14x+2}{2x^2+7x+3}$ c. $\dfrac{6x+2}{3x+4}$ d. $\dfrac{-6x^2+22x+2}{2x^2+7x+3}$ e. $\dfrac{-6x^2+4x+3}{2x^2+7x+3}$

6. What is the measure of an interior angle of a regular octagon?

   a. 45°   b. 67.5°   c. 120°   d. 135°   e. 150°

7. It took 3 teenagers a total of 5 hours to clean the litter from a large park. If 4 teenagers worked at the same rate, how long would it take them to clean the park? Assume all teenagers contribute equally.

   a. 2.40 hours   b. 3.75 hours   c. 4.00 hours   d. 5.25 hours   e. 6.67 hours

8. One DJ charges $20 per hour but is currently running a special which reduces the total fee by $50. A second DJ charges $16 per hour. Jacie planned her party, and noticed that the costs for the two DJs were identical. How long is Jacie's party?

   a. 3.25 hours   b. 5.75 hours   c. 8 hours   d. 9.5 hours   e. 12.5 hours

9. A robotic welding machine needs to move from point (1,3) to point (4, 4). What is the straight line distance between these two points?

   a. 2   b. $\sqrt{7}$ c. $\sqrt{8}$ d. $\sqrt{10}$ e. 4

10. At Kenbrooke High School, 15% of the students have no pets, 23% have 1 pet, 42% have 2 pets, 8% have 3 pets, and the rest have 4 pets. What is the average number of pets per student at Kenbrooke High School?

   a. 1.79   b. 1.94   c. 2   d. 2.31   e. 2.42

11. The value of $\frac{(1-\sqrt{5})(2+\sqrt{5})}{7+\sqrt{5}}$ is equal to:

a. $\dfrac{1+\sqrt{5}}{4}$ b. $\dfrac{4-\sqrt{5}}{11}$ c. $\dfrac{-4-\sqrt{5}}{11}$ d. $\dfrac{8+4\sqrt{5}}{11}$ e. $\dfrac{27+15\sqrt{5}}{44}$

12. How many values of x satisfy the equation $\dfrac{2x^2-10x}{x^2-5x}=x-3$?

a. 0 b. 1 c. 2 d. 3 e. 4

13. The numbers 1, 3, 6, 10, 15, . . . are known as *triangular numbers*. Each triangular number can be expressed as $\dfrac{n(n+1)}{2}$, where n is a natural number. The sum of the two largest triangular numbers less than 500 is:

a. 890 b. 900 c. 935 d. 961 e. 990

14. A regular hexagon is inscribed in a circle. Approximately what portion of the area of the circle lies within the hexagon?

a. 78.3% b. 80.1% c. 82.7% d. 83.3% e. 85.7%

15. What is the sum of all solutions to the equation $|3x+4|=2$?

a. $-\dfrac{8}{3}$ b. $-\dfrac{2}{3}$ c. $-\dfrac{1}{3}$ d. 0 e. $\dfrac{4}{3}$

16. A right triangle has a hypotenuse of 4 and a perimeter of 9. What is the difference between the lengths of the two legs?

a. 0 b. 1 c. $\sqrt{5}$ d. $\sqrt{7}$ e. 5

17. A computer password consists of four different capital letters (A through Z) followed by two different digits (0 through 9). How many different passwords of this type do not contain either a Q or a Z and end in an odd number?

a. 3,825,360 b. 11,476,080 c. 12,476,080 d. 16,588,800 e. 22,952,160

18. A bag contains 2 red chips, 5 blue chips, and some yellow chips. All the chips are identical except for their color. If 1 red, 1 blue, and 1 yellow chip are removed from the bag, the probability of selecting a red chip decreases by 40%. What is the probability of selecting a red chip from the original bag?

a. $\dfrac{2}{15}$ b. $\dfrac{1}{9}$ c. $\dfrac{2}{13}$ d. $\dfrac{3}{10}$ e. $\dfrac{1}{11}$

19. A three-digit <u>odd</u> number, x, has the following properties:
   - The difference between the hundreds and tens digits is the same as the difference between the tens and ones digits.
   - The hundreds digit is greater than the sum of the tens and ones digits.
   - The sum of the three digits is 15, and all three digits are different.

   Which of the following is true?

a. 500 < x < 599 b. 600 < x < 699 c. 700 < x < 799 d. 800 < x < 899 e. 900 < x < 999

20. A company makes rubber balls with a surface area of 1200 in². Each ball is packaged for sale in a cubic box. What is the best estimate of the minimum length of the side of a box needed to hold a ball with one-eighth inch clearance on all sides?

   a. 9.772 inches   b. 10.022 inches   c. 19.544 inches   d. 19.670 inches   e. 19.795 inches

21. An energy company needs to generate 2650 megawatts (MW) of electricity. The cost to generate electricity from solar panels is $7200 for one megawatt (MW) and for coal it costs $3100 for one megawatt (MW). How much of the electricity can be generated from solar panels and still keep the total cost below $9,300,000?

   a. 105.3 MW   b. 264.6 MW   c. 1852.4 MW   d. 2385.4 MW   e. 4271.9 MW

22. The length of a rectangle is 7 feet more than the width. If the length is decreased by 3 feet and the width is increased by 2 feet, the perimeter becomes 32 feet.   Find the area of the original rectangle, in square feet.

   a. 21   b. 35   c. 36   d. 60   e. 63

23. A full radiator has 60% anti-freeze and 40% water. The owner wishes to raise the concentration to 90% anti-freeze. What fraction of the liquid in the radiator should be drained and replaced with pure anti-freeze?

   a. $\dfrac{1}{2}$ b. $\dfrac{2}{3}$ c. $\dfrac{3}{4}$ d. $\dfrac{4}{5}$ e. $\dfrac{5}{6}$

24. When a manufacturer produces 500 calculators, they charge $12.30 each. When producing 750 calculators, they charge $11.20 each. Assume that price is a linear function of the number of calculators produced. What price should be charged for each calculator if 900 are produced?

   a. $9.44   b. $10.10   c. $10.42   d. $10.54   e. $10.83

25. When the diameter of a circle is increased by π units, by how many units is the circumference of the circle increased?

   a. $\dfrac{1}{\pi}$ b. π c. $\dfrac{\pi^2}{2}$ d. 2π e. π^2

26. $\overline{AB}$ is both a diameter of a circle of radius 2 and a side of an equilateral triangle $\triangle ABC$. The circle intersects $\overline{AC}$ at point D and $\overline{BC}$ at point E (see figure below). The length of $\overline{AE}$ is

   a. $\dfrac{3}{2}$ b. $\dfrac{5}{3}$ c. $\sqrt{3}$ d. $2\sqrt{3}$ e. 2

27. A rhombus is inscribed in $\triangle ABC$ such that two of its sides lie on $\overline{AB}$ and $\overline{AC}$ (see figure below). If $AC = 6$ in, $AB = 12$ in, and $BC = 8$ in, then the length of the side of the rhombus, in inches, is

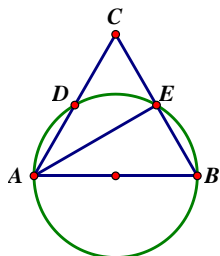   a. 4   b. 4.25   c. 4.5   d. 4.75   e. 5
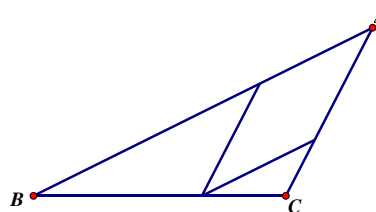
Figure for Problem 26

Figure for Problem 27

28. If 4567^{4567} is multiplied out, the units' digit in the final product would be

a. 1   b. 3   c. 5   d. 7   e. 9

29. Which of the following best describes the quadrilateral with vertices (-1 , 1), (1, -2), (5, 0), and (3, 3)?

a. A rectangle but not a square   b. A square   c. A rhombus but not a square

d. A parallelogram but not a rectangle or a rhombus   e. None of these

30. How many integers are there between 1,000 and 9,999 that have four distinct digits and the absolute value of the difference between the first digit and the last digit is 2?

a. 672   b. 784   c. 840   d. 896   e. 1008

31. Tara paid one-half of her game-show winnings to the government for taxes. She invested one-third of her winnings in Jeff's Copy Shop at 14% interest and one-sixth of her winnings in Kaiser's German Bakery at 12% interest. Tara earned a total of $4000 on these investments in one year. How much did she win on the game show?

a. $6000   b. $30,000   c. $60,000   d. $90,000   e. $120,000

32. Della can scrape the barnacles from a yacht in 10 hours using an electric barnacle scraper. Don can do the same job in 15 hours using a manual barnacle scraper. If Don starts scraping at noon and Della joins him at 3 p.m., then at what time will they finish the job?

a. 6:00 p.m.   b. 6:24 p.m.   c. 6:40 p.m.   d. 7:12 p.m.   e. 7:48 p.m.

33. Suppose that each girl in a math class has 6 more female classmates than male classmates, whereas each boy in the class has twice as many female classmates as male classmates. How many girls are there in the math class?

a. 9   b. 12   c. 16   d. 20   e. 24

34. The frequency table below shows the number of attempts needed for students to successfully complete an obstacle course.

Attempts	1	2	3	4	5	6	7	9	12
Frequency	12	16	14	10	7	5	4	2	1

When comparing the mean, median, and mode of the data, which one of the following is true?

a. median < mean   b. mean < mode   c. mean < median

d. mean = median   e. mean = median = mode

35. Americium-241 is a radioactive element best known for its use in smoke detectors. Scientists know that Americium-241 decays exponentially, and that half of the atoms of Americium-241 decay into other elements every 432 years. Approximately what percent of a collection of Americium-241 will decay in 216 years?

a. 25%   b. 27%   c. 29%

d. 31%   e. 33%