

2011 SCSU MATH CONTEST
9th and 10th Grade Test

DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

1. Four points are on a line segment, as shown (but not shown to scale).



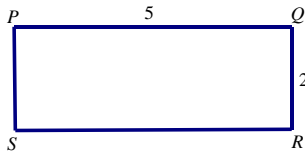
If $AB : AC = 1 : 3$ and $BC : CD = 8 : 5$, then what is $AB : BD$?

- a. 1:7 b. 1:13 c. 3:13 d. 4:13 e. 4:17
2. Two students go on a three-day road trip. On the second day, they travel 2 miles more than half the distance they travelled on the first day. On the third day, they travel three times as far as they did on the second day. If they travelled 1577 miles total, how far did they travel the third day?
- a. 263.5 miles b. 523 miles c. 617 miles d. 784.5 miles e. 790.5 miles
3. ABC is a right triangle with legs $BA = 8$ and $BC = 6$. Point D is between B and C such that $BD = 5$. Point E lies on \overline{BA} extended such that $BE = 12$. Point F is the intersection of \overline{AC} and \overline{DE} . Find the distance from D to F .
- a. $\frac{13}{5}$ b. 3 c. $\frac{13}{4}$ d. $\frac{15}{4}$ e. None of these
4. A right trapezoid has bases of length 3 cm and 8 cm and area 33 cm^2 . What is the perimeter of this trapezoid, to the nearest cm?
- a. 23 cm b. 24 cm c. 25 cm d. 26 cm e. 28 cm
5. What is the units (ones) digit in the product $(5 + 1)(5^3 + 3)(5^6 + 6)(5^{12} + 12)$?
- a. 0 b. 2 c. 4 d. 6 e. 8
6. Assume $w = 2^{129} \times 3^{81} \times 5^{128}$, $x = 2^{127} \times 3^{81} \times 5^{128}$, $y = 2^{126} \times 3^{82} \times 5^{128}$, and $z = 2^{125} \times 3^{82} \times 5^{129}$. What is the order of the four numbers from smallest to largest?
- a. w, x, y, z b. x, w, y, z c. x, w, z, y d. x, y, z, w e. z, y, x, w
7. Three rugs have a combined area of 200 square meters. By overlapping the rugs to cover a floor area of 140 square meters, the area which is covered by exactly two layers of rug is 24 square meters. How many square meters of floor is covered by three layers of rug?
- a. 12 b. 18 c. 24 d. 36 e. 42
8. A student has four exams scores. The average of the first two scores is 50. The average of the second and third is 75 and the average of the third and fourth is 70. What is the average of the first and fourth scores?
- a. 45 b. 50 c. 90 d. 100 e. None of these
9. Find the area of the region bounded by the graphs of $y = ||x| - 5|$ and the graph of $y = 5$.
- a. 10 sq units b. 20 sq units c. 25 sq units d. 50 sq units e. 60 sq units
10. If eggs were x cents a dozen less, you would pay 1 cent less for $x + 1$ eggs than if they were x cents a dozen more. Find x .
- a. 1 b. 2 c. 3 d. 4 e. 5

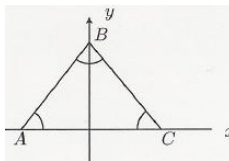
11. A jar contains only red and only yellow jelly beans. If a child eats 1 red jelly bean, $\frac{1}{7}$ of the remaining candies will be red. If instead the child eats 5 yellow jelly beans, $\frac{1}{6}$ of the remaining candies will be red. How many jelly beans are in the jar?
- a. 29 b. 43 c. 71 d. 113 e. None of these
12. Jack needed a new couch and decided to 'rent-to-own' one. He paid a one-time service fee of \$25.00 and rent of \$23.69 per month for 30 months. He then paid an end payment of \$50 after which he owned the couch. Jack could have originally purchased that same couch for \$299.00. What percent of the couch's purchase price, to the nearest percent, did Jack pay in interest in his 'rent-to-own' arrangement?
- a. 92% b. 105% c. 163% d. 262% e. None of these
13. McKenna likes either red or green clothes, but not both. She likes either turtleneck or V-neck sweaters, but not both. McKenna never wears a sweater that is both the color and the type she likes, nor does she wear one that is neither the color nor the type she likes. McKenna wears a red turtleneck. I want to buy a sweater for McKenna that she will wear. Should I buy a red V-neck, a green V-neck, or a green turtleneck?
- a. A red V-neck b. A green V-neck c. A green turtleneck
d. McKenna won't wear any of those sweaters. e. Not enough information is given to answer the question
14. At the start of class all the students are awake. During the class, students fall asleep at the rate of one student every 30 seconds. At the tenth minute, the first student wakes up, and the sleeping students wake up at a rate of one student for every minute. (A student who has awoken can go back to sleep.) If the class has thirty students, the first time all students will be asleep will be how many minutes after class began?
- a. 15 b. 20.5 c. 30 d. 35.5 e. 40.5
15. $50! = (50 \cdot 49 \cdot 48 \cdot \dots \cdot 3 \cdot 2 \cdot 1)$ is a huge number. We observe that 3 divides $50!$, $9 (=3^2)$ divides $50!$, and $27 (=3^3)$ divides $50!$. What is the highest power of 3 that can divide $50!$?
- a. 16 b. 18 c. 19 d. 21 e. 22
16. Suppose x is a two-digit positive integer. Switching the two digits results in a new two-digit number y . If $|x-y|=63$, what is a possible value for $x+y$?
- a. 36 b. 54 c. 66 d. 88 e. 121
17. A circle of radius 10 is centered at O with a point A on its circumference. Chord \overline{BC} is a perpendicular bisector of \overline{OA} . What is the area of the shape enclosed by \overline{BC} and arc BAC ?
- a. $\frac{100}{3}\pi - \frac{25}{2}\sqrt{3}$ b. $\frac{100}{3}\pi - 25\sqrt{3}$ c. $\frac{100}{3}\pi - \frac{25}{2}$ d. $\frac{100}{3}\pi - 25$ e. None of these
18. An ahnentafel is a family tree with a special numbering system. You are placed in position 1. Your father and mother are in positions 2 and 3 respectively and are one generation back from you. Your father's father and mother are 4 and 5, and your mother's father and mother are 6 and 7 (and are two generations back from you), and so on. Someone just told you that ancestor number 4122 came to America on the Mayflower. Find that person's sex, how many generations back from you he or she lived, and whether that person comes from your father's side of the family or your mother's.
- a. Male, 13, father's b. Female, 13, mother's c. Male, 12, father's
d. Female, 12, father's e. Male, 11, mother's

19. The average age of a group of teachers and professors is 40. The teachers' average age is 35 years old and the professors' is 50 years old. What is the ratio of the number of teachers to professors?
- a. 1:3 b. 1:2 c. 1:1 d. 2:1 e. 3:1
20. Suppose that a population follows the recursive logistic model where $P_{n+1} = 3.2P_n(1 - P_n)$. For example, if $P_0 = 0.25$, then $P_1 = 3.2(.25)(1 - .25) = 0.6$. Find the average of P_4 and P_5 if $P_0 = 0.4$. Round your answer to the nearest hundredth.
- a. 0.32 b. 0.54 c. 0.67 d. 0.78 e. 0.81
21. Four children are playing with marbles. At the end of the day one child has 4 less than half the marbles, the second child has 6 more than one fifth of the marbles, the third child has one third of what the first child has and the fourth child has 1 less than the third child. How many marbles are there?
- a. 25 b. 50 c. 75 d. 100 e. None of these
22. An urn contains 3 pennies, 3 nickels, 3 dimes and 3 quarters. Jeremy reaches in and randomly selects 3 coins. What is the probability that the three coins will have a value less than 50 cents? Assume that all coins are equally likely to be selected.
- a. $\frac{3}{4}$ b. $\frac{23}{24}$ c. $\frac{48}{55}$ d. $\frac{131}{132}$ e. $\frac{201}{220}$
23. If $x - 2\sqrt{x} + 1 + x^2 + 4y^2 - 4xy = 0$, then $\frac{1}{x} + \frac{1}{y} =$
- a. 3 b. 4 c. 5 d. 6 e. None of Above

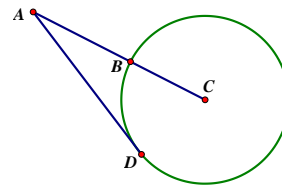
The three diagrams below pertain to questions 24-26, respectively.



Question 24



Question 25



Question 26

24. On a rectangular table 5 units long and 2 units wide (see figure above left), a ball is rolled from point P at an angle of 45° to \overline{PQ} and first bounces off \overline{SR} . The ball continues to bounce off the sides at 45° until it reaches one of the corners. How many bounces of the ball are required until the ball reaches a corner?
- a. 2 b. 3 c. 4 d. 5 e. 6
25. Suppose the triangle $\triangle ABC$ shown in the diagram (see figure above center) is an equilateral triangle with $\overline{AC} = 2$. Write the equation of the line through B and C .
- a. $y = -\sqrt{3}x + \sqrt{3}$ b. $y = -2x + \sqrt{3}$ c. $y = -\sqrt{3}x + 1$ d. $y = -x + \sqrt{3}$ e. $y = -\frac{\sqrt{3}}{2}x + 2$
26. In the diagram (see figure above right), C is the center of the circle and \overline{AD} is tangent to the circle at D . \overline{AC} intersects the circle at B . If $AD = 10$ and $AB = 7$, the radius of the circle is:
- a. $\frac{\sqrt{151} - 7}{2}$ b. $\sqrt{14}$ c. $\frac{51}{14}$ d. $\frac{\sqrt{51}}{2}$ e. $\frac{7}{2}$

27. Peter has only quarters, dimes and nickels in his piggy bank. There are 60 coins altogether in the bank. If the quarters were dimes and the dimes were quarters, the total value of the coins would be increased by 90 cents. Also, if the nickels were dimes and the dimes were nickels, the total value of the coins would be increased by 15 cents. What is the total value of the coins in Peter's piggy bank?
- a. \$6.00 b. \$6.05 c. \$7.00 d. \$7.05 e. \$8.00
28. The level of a prescription drug in the human body over time can be found using the formula $L = \frac{D}{1 - (0.5)^{\frac{n}{h}}}$, where D is the amount taken every n hours and h is the drug's half-life in hours. If a doctor wants the level of drug to build up to a level of 5.4 milligrams in a patient taking 2.5 milligram doses of a drug with a half-life of 9 hours, approximately how often should the doses be taken?
- a. Every 6 hours b. Every 8 hours c. Every 10 hours d. Every 12 hours e. Every 14 hours
29. On a 100 point test, the mean of six scores is 85. Find the sum of the lowest possible score and the highest possible standard deviation.
- a. 43.54 b. 51.73 c. 66.67 d. 78.24 e. 92.16
30. In simplest form, $a^{-1} - [b^2(2 - 3)^{-1}]^{-1}$ is
- a. $\frac{a - b^2}{ab^2}$ b. $\frac{b^2 + a}{ab^2}$ c. $\frac{b^2 - a}{a^2}$ d. $\frac{a^2 - b}{ba^2}$ e. $\frac{a^2 + b}{ba^2}$
31. Mary and Gary graduated from college together. Gary became a teacher and earned half of what Mary earned for five years. Mary spent $\frac{1}{3}$ of her money; Gary spent $\frac{1}{4}$ of his. Gary has 81,000 dollars after five years. How much does Mary have?
- a. \$72,000 b. \$108,000 c. \$144,000 d. \$160,000 e. \$216,000
32. At the LowCost Sandwich Shop, 3 sandwiches, 7 cups of coffee, and 1 donut cost \$31.50. Buying 4 sandwiches, 10 cups of coffee, and one donut cost \$43.00. How much would 1 sandwich, 1 cup of coffee, and 1 donut cost?
- a. \$8.50 b. \$9.00 c. \$9.50 d. \$10.00 e. \$10.50
33. A girl writes 4 letters to 4 friends, and addresses 4 corresponding envelopes. Then she randomly stuffs the letters in the envelopes. What is the probability that none of the letters will be put into the right envelopes?
- a. $\frac{2}{13}$ b. $\frac{1}{12}$ c. $\frac{1}{4}$ d. $\frac{3}{8}$ e. None of these
34. Suppose that for all $x > 0$ we have $f(2x) = \frac{2}{2+x}$. What is $2f(x)$?
- a. $\frac{2}{1+x}$ b. $\frac{2}{2+x}$ c. $\frac{4}{1+x}$ d. $\frac{4}{2+x}$ e. $\frac{8}{4+x}$
35. A man is in Rome, and wants to go to Montpellier. And he goes there in 11 days, neither more nor less. And another man is at Montpellier, and wants to go to Rome. And he goes there in 9 days, neither more nor less. Now they leave at exactly the same time, and travel precisely toward one another. In how many days will they meet?
- a. 3.75 days b. 4.95 days c. 5 days d. 5.25 days e. 5.35 days