

**2010 SCSU MATH CONTEST**  
**9<sup>th</sup> and 10<sup>th</sup> Grade Test**

DIRECTIONS: Select the BEST completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. If the sum of the first  $N$  counting numbers is  $15N$ , find  $N$ .  
a. 25                      b. 27                      c. 29                      d. 31                      e. 33
2. Joe has quarters, dimes, and nickels worth \$7.20. He has five times as many dimes as quarters and four more nickels than dimes. How many coins does Joe have?  
a. 80                      b. 81                      c. 82                      d. 83                      e. 84
3. Before she took her last exam, the average of Tara's exam scores was 89. If she scores 97 on the last exam, her average for all the exams will be 90. If Tara scores 73 on the last exam, what will her final average be for all exams?  
a. 84                      b. 85                      c. 86                      d. 87                      e. 88
4. Larry and Lisa share a piece of land. The ratio of the area of Larry's portion to the area of Lisa's portion is 3:2. They each grow wheat and beans on their piece of land. The entire piece of land is covered by wheat and beans in the ratio of 7:3. On Larry's portion of the land, the ratio of wheat to beans is 4:1. What is the ratio of wheat to beans for Lisa's portion?  
a. 4:3                      b. 5:3                      c. 7:5                      d. 9:7                      e. 11:9
5. A right triangular prism has its edge lengths in the ratio 3:4:5:10. The volume of this prism is 202.5 cubic units. What is the actual length of the longest edge?  
a. 10                      b. 15                      c. 20                      d. 25                      e. 30
6. Let  $A$ ,  $B$ , and  $C$  be integers between and including 0 and 9. If the 7-digit number  $2A5B4C6$  is a multiple of 12,  $A + B + C$  CANNOT equal  
a. 1                      b. 7                      c. 13                      d. 17                      e. 19
7. Express  $0.\overline{1} + 0.\overline{12} + 0.\overline{123}$  as a fraction in simplest form.  
a.  $\frac{43}{99}$                       b.  $\frac{56}{121}$                       c.  $\frac{73}{549}$                       d.  $\frac{256}{999}$                       e.  $\frac{434}{1221}$
8. Tot is now half as old as Teenager was when Tot was a third as old as Teenager is now. Assume that each age is a whole number and that Teenager is a teenager. What is the sum of Teenager's and Tot's ages?  
a. 20                      b. 22                      c. 24                      d. 26                      e. 28
9. The point  $(1, 2)$  is reflected over the  $x$ -axis and this image is then reflected over the line  $y = x$ . What are the coordinates of the point  $(1, 2)$  after these transformations?  
a.  $(2,1)$                       b.  $(-2,1)$                       c.  $(1,-2)$                       d.  $(1,2)$                       e.  $(-1,-2)$

10. On a small island, all the inhabitants are Derfs, Enajs, and/or Sivads. All Derfs are Enajs. One-third of all Enajs are Derfs. Half of all Sivads are Enajs. One Sivad is a Derf. Eight Sivads are Enajs. The number of Enjas is 90. How many Enajs are neither Derfs nor Sivads?

- a. 38                      b. 44                      c. 52                      d. 53                      e. 60

11. A regular polygon has interior angles of measure 162 degrees. How many sides does this polygon have?

- a. 18                      b. 19                      c. 20                      d. 21                      e. 22

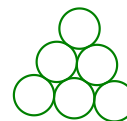
12. Consider these four statements.

- i.  $A + B \leq |A + B|$                       ii.  $\sqrt{A^2} = A$                       iii.  $\sqrt{A} \leq A$                       iv.  $|A + B| \leq |A| + |B|$

How many of these statements are true for all real numbers A and B?

- a. 0                      b. 1                      c. 2                      d. 3                      e. 4

13. Six pipes, each with a radius  $a$ , are stacked as shown in the figure. Find the height of the pile in terms of  $a$ .



- a.  $6a$                       b.  $3a$                       c.  $2a\sqrt{3}$                       d.  $(4\sqrt{3} + 2)a$                       e.  $(2\sqrt{3} + 2)a$

14. A mathematician lays out six cards as shown below. Each card has a letter on one side and a positive integer on the other. You are told that any card with a prime number on one side must have a vowel on the other side. What is the minimum number of cards you must flip over to verify that none of the cards violate the rule?

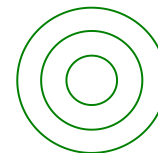


- a. 1                      b. 2                      c. 3                      d. 4                      e. 5

15. How many positive factors does 10,000 have?

- a. 25                      b. 26                      c. 27                      d. 28                      e. 29

16. Concentric circles are constructed so that the innermost circle is of radius 1 unit, the second is of radius 2, the third is of radius 3, etc. This is continued for  $n$  circles. What is the area of the outermost ring?



- a.  $(2n - 1)\pi$                       b.  $n^2\pi$                       c.  $(n^2 - 1)\pi$   
 d.  $(n^2 - n)\pi$                       e.  $(n^2 - 2n + 1)\pi$

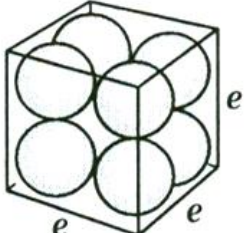
17. A sample of ten numbers has a mean of nine, a maximum of fourteen, and a minimum of four. What is the highest value that the standard deviation,  $s$ , could have?

- a.  $1 \leq s < 2$                       b.  $2 \leq s < 3$                       c.  $3 \leq s < 4$                       d.  $4 \leq s < 5$                       e.  $5 \leq s < 6$

18. A teacher said his class had 100 students, 24 of whom were boys and 32 of whom were girls. What number base did this teacher use in this statement?

- a. 5                      b. 6                      c. 7                      d. 8                      e. 9



26. Each time a customer visits a restaurant he/she is given a game card. Suppose the probability of winning a prize with the game card is 0.1. What is the probability, to the nearest hundredth, that a customer will win a prize in five or fewer visits?
- a. 0.40                      b. 0.41                      c. 0.45                      d. 0.52                      e. 0.62
27. The sum of two numbers is 8 and the sum of their cubes is 20. What is the sum of the squares of these two numbers?
- a. 10                          b. 13                          c. 17                          d. 21                          e. 23
28. Consider the family of quadratic equations given by  $y = (m^2 - m + 1)x^2 - (2m)x + 1$ , where  $m$  is any real number. For what values of  $m$  will the graphs of the equations lie above the  $x$ -axis?
- a.  $(-1, \infty)$                       b.  $(\frac{3}{2}, \infty)$                       c.  $(-\infty, 1)$                       d.  $(-\infty, 1) \cup (\frac{3}{2}, \infty)$                       e.  $(-\infty, \frac{3}{2}) \cup (1, \infty)$
29. Marissa found all positive solutions to:  $(\log_{10} x^2)^2 = \log_{10}(x^4)$ . She then added these numbers. What sum should Marissa get?
- a. 7                              b. 8                              c. 9                              d. 10                              e. 11
30. Eight congruent spheres are packed into a cube of length  $e$  so that each of the spheres is tangent to three faces of the cube and to three other spheres. What is the ratio of the total volume of the eight spheres to the volume of the cube?
- a.  $\pi:2$                               b.  $\pi:3$                               c.  $\pi:4$   
d.  $\pi:6$                               e.  $\pi:8$
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31. The number  $2^{48} - 1$  has two divisors between 50 and 75. Find the sum of the two numbers.
- a. 113                          b. 118                          c. 123                          d. 128                          e. 133
32. Find the area of a rhombus that has one side of length 10 and diagonals that differ by 4.
- a. 96                              b. 100                              c. 104                              d. 108                              e. 112
33. If  $\log_{15} 5 = a$ , express  $\log_{15} 9$  in terms of  $a$ .
- a.  $\frac{9}{5}a$                               b.  $2a$                               c.  $1 - \frac{9}{5}a$                               d.  $2 - 2a$                               e.  $2 - \frac{9}{5}a$
34. Four positive integers sum to 125. If you increase the first of these numbers by 4, decrease the second by 4, multiply the third by 4, and divide the fourth by 4, you produce four equal numbers. What is the difference between the largest and smallest of these four numbers?
- a. 55                              b. 60                              c. 65                              d. 70                              e. 75
35. When Rachel divides her favorite number by 7, she gets a remainder of 5. What will the remainder be if she multiplies her favorite number by 5 and then divides by 7?
- a. 2                                  b. 3                                  c. 4                                  d. 5                                  e. 6