

2009 SCSU MATH Contest
9th and 10th Grade Test

- Let $f(x) = x^2 + x - 6$. For what values of t does $f(t - 5) = 0$?
 - 6 and 1
 - 3 and 2
 - 2 and 3
 - 0 and 5
 - 2 and 7
- Jane gets a 10% raise every year. To the nearest percent, how much has her salary increased after four such raises?
 - 40%
 - 46%
 - 54%
 - 63%
 - 66%
- The equation $x^2 - x = a^2 + a$ has exactly one solution. The number a must be
 - $-\frac{1}{2}$
 - $-\frac{1}{4}$
 - 0
 - $\frac{1}{2}$
 - 1
- The first four terms of an infinite sequence are $-1 + 2a$, $3 - 4a$, $-5 + 6a$, $7 - 8a$. What is the 2009th term of this sequence?
 - $-4017 + 4018a$
 - $4017 - 4018a$
 - $-4019 + 4018a$
 - $-4019 + 4020a$
 - $4019 - 4020a$
- A woman has nine close friends. In how many ways can she invite exactly six of these friends to dinner, under the condition that two of her friends are not speaking and will not attend together?
 - 42
 - 49
 - 63
 - 72
 - 82
- 2137_{eight} is what numeral in base 5?
 - 2234_{five}
 - 3410.2_{five}
 - 10432_{five}
 - 13434_{five}
 - 20213_{five}
- Triangle ABC is defined by the vertices $A(0, -6)$, $B(1, 0)$, and $C(6, -7)$. What type of triangle is ABC?
 - Scalene Acute
 - Scalene Right
 - Scalene Obtuse
 - Isosceles Acute
 - Isosceles Right
- Find the y -intercept of the perpendicular bisector of \overline{AB} if A is $(-4, -3)$ and B is $(2, 5)$.
 - $-\frac{16}{3}$
 - $\frac{1}{4}$
 - 2
 - 4
 - $\frac{19}{3}$
- Assume $a \geq b \geq c \geq d$. If the mean of a, b, c , and d is 8 and the median of a, b, c , and d is 5, then $a + d$ is
 - 6
 - 11
 - 22
 - 27
 - Impossible to Determine
- If three fair dice are tossed and the product of the numbers that appear is even, what is the probability that the sum of the numbers is also even?
 - $\frac{2}{5}$
 - $\frac{3}{5}$
 - $\frac{4}{7}$
 - $\frac{5}{7}$
 - $\frac{7}{9}$

20. Define $a + b = \frac{ab}{a+b}$ and $a - b = \frac{1}{a} + \frac{1}{b}$, where a, b , and $a + b$ are nonzero real numbers.

If $2(k - 3) = 0$, then k is

- a. -5 b. $-\frac{7}{2}$ c. $-\frac{6}{5}$ d. $-\frac{4}{3}$ e. $\frac{2}{3}$

21. Function f is defined on the interval $[0,1]$ by $f(x) = \begin{cases} x + \frac{1}{2}, & 0 \leq x < \frac{1}{2} \\ 2 - 2x, & \frac{1}{2} \leq x \leq 1 \end{cases}$

Let f^n denote the n th iteration of f , so $f^2(x) = f(f(x))$, $f^3(x) = f(f(f(x)))$, What is $f^{2009}\left(\frac{1}{2}\right)$?

- a. 0 b. $\frac{1}{4}$ c. $\frac{1}{2}$ d. $\frac{3}{4}$ e. 1

22. Mary had a coin purse with fifty coins (pennies, nickels, dimes, and quarters) that total exactly \$1.00. She has at least one of each coin. Unfortunately, while counting her change, she dropped one coin. What is the probability that the dropped coin was a penny?

- a. 0.5 b. 0.6 c. 0.7 d. 0.8 e. 0.9

23. Two numbers have a sum of 8 and a product of 11. What is the sum of their reciprocals?

- a. $\frac{2}{3}$ b. $\frac{4}{5}$ c. $\frac{6}{7}$ d. $\frac{7}{9}$ e. $\frac{8}{11}$

24. Three girls and two boys are on the Math Team. [Note: Names do not indicate gender.]

Two students wear white shirts; three wear black shirts.

Miken and Carry wear shirts of different colors.

Bari and Jamie wear shirts of the same color.

Pita and Carry are the same gender.

Jamie and Miken are of different genders.

The boy in the white shirt scored the most points. What is his name?

- a. Bari b. Carry c. Jamie d. Miken e. Pita

25. Working alone, pump A can fill a pool in 8 hours, while pump B can fill it in 10 hours. The two pumps are turned on at the same time and run until the pool is 75% full. Pump A then stops working, but pump B continues until the pool is filled. How long does it take to fill the empty pool?

- a. $\frac{10}{3}$ hours b. $\frac{16}{3}$ hours c. $\frac{23}{6}$ hours d. $\frac{29}{6}$ hours e. $\frac{35}{6}$ hours

26. How many distinct 3-digit numbers have a digit sum of 8?

- a. 36 b. 42 c. 45 d. 46 e. 50

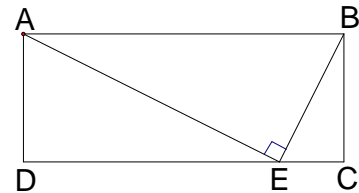
27. The Queens Midtown Tunnel is 6414 feet long. A train that is 186 feet long is traveling at a constant rate of 40 miles per hour. Compute the number of minutes from the time that the train enters the tunnel until it clears the tunnel.

- a. 1.25 min b. 1.6 min c. 1.875 min d. 2.1 min e. 2.375 min

28. If the diagonal of a rectangle has length 7 feet and the rectangle's area is 16 square feet, what is the perimeter of the rectangle in feet?
- a. 10 feet b. 15 feet c. 18 feet d. 23 feet e. 26 feet

29. How many ordered triples (x, y, z) satisfy $x^2 + y^2 + z^2 < 4$, where $x, y,$ and z are integers?
- a. 7 b. 13 c. 19 d. 27 e. 31

30. In the rectangle shown at right, $AD = 8$ and $CD = 20$.
 E is chosen on CD such that $m \angle AEB = 90^\circ$.
 What is the product of CE and ED ?



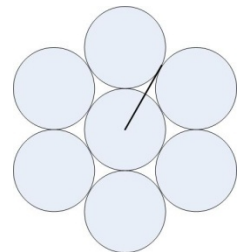
- a. 30 b. 42 c. 56 d. 64 e. 78

31. Mr. Locksmith has \$1001, and his son has \$1. Mr. Locksmith gives $\frac{1}{4}$ of his money to his son, who, at the same time gives $\frac{1}{4}$ of his money to his father. They repeat this procedure until the difference between their money is less than fifty dollars. How many times will they repeat the procedure?
- a. 3 b. 4 c. 5 d. 6 e. 7 or more

32. How many numbers from 1 to 1 million, inclusive, are NOT perfect squares or perfect cubes?
- a. 998,000 b. 998,830 c. 998,900 d. 998,910 e. 999,000

33. Six circles of radius r fit exactly (no gaps, no overlaps) around a central circle of radius r , as shown. What is the length, in terms of r , of the line segment joining the center of the central circle to the point where two outer circles touch?

- a. $\sqrt{5}r$ b. $\sqrt{3}r$ c. $2r$ d. $3r$ e. $5r$

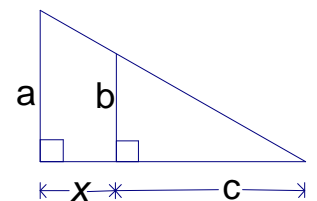


34. Find the distance between the two points of intersection of $y = x^2 - x$ and $y = 3x + 5$.

- a. $\sqrt{10}$ b. 6 c. $6\sqrt{10}$ d. 8 e. $2\sqrt{130}$

35. Which expression represents the length of the segment x in the given figure?

- a. ca b. $ca - 1$ c. $\frac{ca}{b} - 1$
 d. $\frac{c(a-b)}{b}$ e. $\frac{c(a+b)}{b}$



36. Each letter in the problem to the right represents a different digit.
 All digits in *NINE* are odd. What is the smallest possible value of *NINE*?

$$\begin{array}{r} \text{FOUR} \\ + \text{FIVE} \\ \hline \text{NINE} \end{array}$$

- a. 3135 b. 3537 c. 5153 d. 5351 e. 7173