2014 SCSU MATH CONTEST
7TH and 8th Grade Test

DIRECTIONS: Select the BEST completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. A computer password consists of three lowercase letters of the alphabet followed by two digits (0 – 9).
    How many different passwords are possible if each of the three letters is a different vowel (a, e, i, o, u)?

A.   120
B.   5400
C.   6000
D.   12,500
E.   None of these

2. What fraction of 2 cubic yards is 4 cubic feet?

A. $\frac{2}{27}$
B. $\frac{1}{6}$
C. $\frac{2}{9}$
D. $\frac{1}{2}$
E. $\frac{2}{3}$

3. If $a + 1 = b - 2 = c + 3 = d - 4,$ which of the four quantities $a, b, c,$ or d is the largest?

A. a
B. b
C. c
D. d
E.   Impossible to determine

4. If 14 mg of gold costs $0.70, then 3 g of gold would cost:

A. $0.15
B. $15
C. $20
D. $150
E. $200

5. A mathematics competition consists of 25 questions.   The points are awarded as follows:

   Each correct answer: 6 points   *Each wrong answer: 0 points*   *Each answer left blank: 2 points*

   The number of points for a student who answers 16 questions with 12 correct and 4 wrong answers is

A.   74
B.   80
C.   84
D.   90
E.   96

6. In the figure, ΔABC has a right angle at C and ∠A = 20°. If BD
    is the bisector of ∠ABC, find ∠BDC.
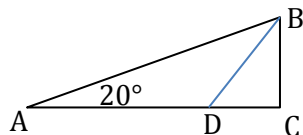
A.   35°
B.   55°
C.   70°
D.   80°
E.   125°

7. In an amazing snapshot, Andre the Giant and the elusive yeti are captured on film together. The yeti's shadow measures 15 mm on the photograph while Andre's measures 9 mm. With your keen math and internet skills, you use the fact that Andre is 7 feet 4 inches tall to determine that the yeti's height to the nearest inch is:

A.   11 ft 7 in.
B.   12 ft 3 in.
C.   12 ft 9 in.
D.   13 ft 4 in.
E.   13 ft 10 in.

8. Alex built a snowman using three snowballs, one small, one medium and one large, with diameters in the ratio 2:3:7. Suppose the three snowballs were perfectly spherical and stacked vertically, one on top of the other, as shown, with adjacent snowballs sharing a single point of tangency. If the diameter of the medium snowball was 18 inches, what was the height, in feet, of the snowman Alex built?

A.   2 ft
B.   3 ft
C.   5 ft
D.   6 ft
E.   7 ft

9. Three identical balls are marked 1, 2, and 3 and placed in a basket. One ball is drawn, its number is recorded, and the ball is returned to the basket. After repeating this process twice more, the sum of the three numbers recorded is 6. What is the probability that the ball numbered 2 was drawn in all three tries?

A. $\frac{1}{27}$
B. $\frac{1}{9}$
C. $\frac{1}{7}$
D. $\frac{1}{6}$
E. $\frac{1}{3}$

10. Abby, Ben, and Chuck are comparing the amount of lemonade each has.

Each person's glass is represented in the figure to the right.

They each make a statement. Who is correct?

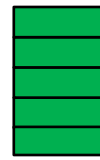Abby: "I have 60% of what Ben has." Ben: "I have 20% less than what Chuck has." Chuck: "I have 25% more than what Ben has."

Abby's glass   Ben's glass   Chuck's glass

A. Only Abby   B. Only Ben   C. Only Chuck   D. Only Abby and Ben   E. Only Ben and Chuck

11. The straight line distance from Fairfield to Pleasantville is 35 miles.
The straight line distance from Pleasantville to Happytown is 12 miles.
Let d represent the straight line distance from Fairfield to Happytown.
Which of the statements to the right are TRUE?

I. d could be 40 miles
II. d could be 25 miles
III. d could be 19 miles

A. All of them   B. None of them   C. II and III only   D. I only   E. I and II only

12. Evaluate $p^3 - r^2 \div t$ for $p = -2, r = -4,$ and $t = 2$.

A. -16 B. -12 C. -4 D. 0   E. 4

13. Among the 500 spectators at a basketball game, 30% were <u>not</u> students. Among the students, 30% were sophomores. Among the sophomores, 60% were males. How many female sophomores were spectators at the game?

A. 18   B. 27   C. 42   D. 54   E. 63

14. A painter has finished painting $\frac{2}{3}$ of a room by 2:00 PM and $\frac{3}{4}$ of the same room by 2:30 PM. At this rate, when does he finish painting the room?

A. 3:30 pm   B. 4:00 pm   C. 4:30 pm   D. 5:00 pm   E. 5:30 pm

15. If the area of the square shown in the figure to the right is 1 square unit, then the area of the irregular polygon on the right is _____ square units.
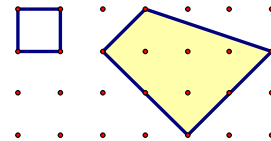
A. 6   B. 8   C. 10   D. 12   E. None of these

16. Suppose $a < -1$ and $0 < b < 1$, which of the following numbers is the smallest?

A. $-\frac{1}{a}$ B. $-a$ C. $|a|$ D. $-b$ E. $-\frac{1}{b}$

17. At 1:45 a.m. on an October day, the temperature in Duluth is 50°F and is falling at the rate of 10°F per hour, and the temperature in St. Cloud is 72.5°F and is falling at the rate of 15°F per hour.
At what time will the temperatures of the two cities be the same?

A. 5:30 am   B. 5:45 am   C. 6:00 am   D. 6:15 am   E. None of these

18. Sam tells lies on Mondays, Tuesdays and Wednesdays, and he tells the truth on all other days. Kate tells lies on Thursdays, Fridays and Saturdays, and she tells the truth on all other days. If Sam says "Tomorrow is a lying day for me" and Kate says "Tomorrow is a lying day for me too," what day of the week is it?

A. Monday   B. Wednesday   C. Thursday   D. Saturday   E. None of these

19. In a coordinate system, a square has two vertices at $(2,2)$ and $(2,-2)$. How many such squares are possible?

A. 1   B. 2   C. 3   D. 4   E. 5

20. 8 lb. of feathers and 2 oz. of gold cost \$932. 14 lb. of feathers and 3 oz. of gold cost \$1402. Find the cost of 5 lb. of feathers and 5 oz. of gold.

A. \$1167   B. \$2300   C. \$2350   D. \$2837.37   E. \$7010

21. Find the equation of the line containing the point $(12,-7)$ and perpendicular to the line $y = \frac{3}{4}x - 7$.

A. $y = -\frac{4}{3}x + 9$ B. $y = -\frac{4}{3}x - 7$ C. $y = \frac{4}{3}x - 23$ D. $y = \frac{3}{4}x - 16$ E. $y = -\frac{3}{4}x + 2$

22. Ten students took a quiz.
The two lowest quiz scores are missing.
The sum of the mean, median, and mode of the <u>ten</u> scores is 28.3.
Find the mean of the two missing scores.

___, ___, 7, 8, 9, 10, 10, 10, 10, 14

A. 3   B. 3.5   C. 4   D. 4.5   E. 5

23. Two squares are positioned as shown.
The smaller square has side length 1 and the larger square has side length 7.
Find the length of AB.
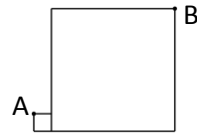
A. $\sqrt{72}$ B. $\sqrt{85}$ C. 10   D. $\sqrt{113}$ E. 14

24. On a remote tropical island, bartering is used in place of currency. If 10 bananas = 3 coconuts, 2 coconuts = 3 pineapples, and 18 pineapples buys one night in a motel, then how many bananas are needed to buy four nights in a motel?

A. 14   B. 36   C. 40   D. 144   E. 160

25. Simplify $9^{30} + 9^{30} + 9^{30}$

A. 9^{90} B. 9^{33} C. 3^{23} D. 3^{61} E. 3^{63}

26. A drawer contains 100 red socks, 80 green socks, 60 blue socks, and 40 black socks. You select socks one at a time from the drawer but are unable to see the colors of the socks. What is the fewest number of socks that must be selected to guarantee that the selection contains at least 10 pairs? (A pair of socks is two socks of the same color.)

A. 20   B. 21   C. 22   D. 23   E. 24

27. Consider the product $\left(1+\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\cdots\left(1+\frac{1}{n}\right)\left(1-\frac{1}{n+1}\right)\cdots\left(1+\frac{1}{48}\right)\left(1-\frac{1}{49}\right)\left(1+\frac{1}{50}\right)$.
The product of these 49 factors is:

A. $\frac{1224}{1225}$ B. 1   C. $\frac{51}{50}$ D. $\frac{153}{100}$ E. None of these

28. A farmer intends to use 240 meters of fence to construct rectangular pens with length L and width W. He plans to build the boundary fences plus one internal dividing fence (see the diagram). If he lets the width be 40 meters, the total area enclosed will be A square meters. If he lets the width be 48 meters, the total area enclosed will be B square meters. The difference $A - B$ (in square meters) equals
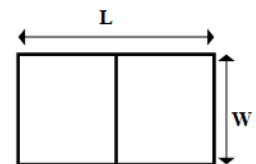
A. 96   B. −96   C. 256   D. −256   E. 0

3

29. What are the last two digits of 7^{2014}?

A. 01   B. 07   C. 14   D. 43   E. 49

30. For numbers a and b, the notation $a\#b$ means $2a + b^2 + ab$. If $x\#(-1) = 8$, determine the value of x.

A. 3   B 7   C. 9   D. 25   E. 54

31. Line segments AF, BG, CH, DI, and EJ intersect at a common point O, as shown.
Segments AB, CD, EF, GH, and IJ then form five triangles (ABO, CDO, EFO, GHO, and IJO).
The sum of the measures of angles A, B, C, D, E, F, G, H, I, and J is:
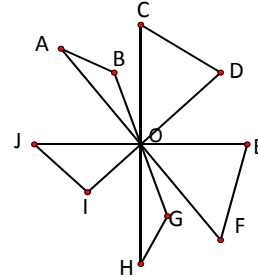
A. 360°   B. 720°   C. 810°   D. 900°   E. Impossible to determine

32. If you add 7 zeros to the binary number 11011, you end up with the binary number 110110000000.
This would be equivalent to multiplying the original number by which of the following base 16 numbers:

A. 30_{16} B. 40_{16} C. 60_{16} D. 70_{16} E. 80_{16}

33. Rectangle ABCD is partitioned into seven congruent rectangles as shown in the figure. The area of rectangle ABCD is 84 square centimeters. What is the perimeter of one of the small rectangles?
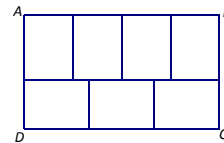
A. 7 cm   B. 9 cm   C. 14 cm   D. 19 cm   E. 38 cm

34. Anna, Brett, and Cassie start painting a fence at the same time. Each painter works at a constant rate.
Working alone, Anna could paint the whole fence in 10 hours, Brett could paint the fence in 20 hours, and Cassie could paint the fence in 30 hours.
Anna paints for 4 hours and quits. Brett continues to paint for an additional hour and then quits.
Cassie finishes painting the fence by herself.
What is the total time Cassie spends painting the fence?

A. 10 hours   B. 10.5 hours   C. 11 hours   D. 20.5 hours   E. None of these

35. Layla added 2 tablespoons of cream to $1\frac{3}{4}$ cups of black coffee. She drank $\frac{1}{4}$ cup of this mixture and decided to add 1 more tablespoon of cream. After mixing it thoroughly, she tasted her drink again and judged it "perfect".
To the nearest tenth, what percent of the "perfect" coffee drink is cream? (1 cup = 16 tablespoons)

A. 6.7%   B. 8.3%   C. 9.4%   D. 10.1%   E. 11.4%

36. When freshly picked, berries are 90% water and 10% pulp, measured by weight. After sitting for a week, part of the water evaporates and the berries become 84% water and 16% pulp. If 40 pounds of freshly picked berries sit for a week, how much will they weigh?

A. 25 pounds   B. 30.5 pounds   C. 33 pounds   D. 33.6 pounds   E. 36 pounds