2025 SCSU MATH CONTEST 11^{th} and 12^{th} **GRADE**

DIRECTIONS: Select the <u>BEST</u> completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

- 1. Yogi and Boo-Boo together can eat a blueberry pie in two minutes. Yogi alone can eat a whole blueberry pie in three minutes. How many minutes would it take Boo-Boo alone to eat a whole blueberry pie?
 - (a) 1.0 (b) 1.5 (c) 2.5 (d) 4.0 (e) 6.0
- 2. How many positive integers divide 2025 with no remainder?
- (a) 12 (b) 13 (c) 14 (d) 15 (e) 16
- 3. In the figure below left, find the length of \overline{CD} . (The figure is NOT to scale.)
 - (a) 2 (b) 3 (c) $3\frac{1}{3}$ (d) 4 (e) 5



4. Because of a construction project in a large circular hall at a state capitol building, a contractor needs to rope off a circular area in the center of the hall, allowing a ring three feet wide, around the roped-off area, for visitors to stand in. (See the illustration above right, which is not to scale.) The diameter of the whole room is 96 feet.

To the nearest *TENTH* of a square foot, what is the floor area of the ring?

- (a) 408.6 (b) 527.0 (c) 679.6 (d) 876.5 (e) 995.4
- 5. How many different (distinguishable) ways are there to rearrange the letters in the word STATISTICS?

(a) 50,400	(b) 209,720	(c) 872,200	(d) 3.628.800	(e) 7.200.000
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6. You bicycle to and from school every day. Your route to school is uphill, and your speed is a constant 10 miles per hour. Your route home is downhill, and your speed is a constant 30 miles per hour. What is your average speed (in miles per hour) over the whole route, to *and* from school?

- (a) 12 (b) 15 (c) 18 (d) 20 (e) 22.5
- 7. What is the sum of all the positive odd integers less than 100?
 - (a) 2500 (b) 3333 (c) 3650 (d) 4208 (e) 5000
- 8. Suppose that A, B, and C are positive numbers such that the mean of A and 2B is 7 and the mean of A and 2C is 8.
 What is the mean of A, B, and C?
 - (a) 3 (b) 4 (c) 5 (d) 6 (e) 9

 Sven is a runne miles per hour. at 3 miles per h 	r, and Ole is a walker. E At the same starting tim our. St. Paul and St. Cle	arly one morning, Sven l ne, Ole begins walking fr oud are 77 miles apart.	pegins running from St. (om St. Paul to St. Cloud	Cloud to St. Paul at 7.2 I, along the same route,		
When Sven and Ole meet en route, how far, to the nearest TENTH of a mile, will Ole have walked?						
(a) 18.4	(b) 19.7	(c) 21.0	(d) 21.4	(e) 22.6		
10. Trina has 24 coins, all dimes and nickels, worth between \$1.72 and \$2.11. What is the smallest number of dimes she could have?						
(a) 10	(b) 11	(c) 15	(d) 18	(e) 19		
11. What is the largest possible AREA, in square inches, of a right triangle with hypotenuse of length 5 inches?						
(a) 5.5	(b) 5.725	(c) 6	(d) 6.25	(e) 6.375		
12. Suppose that x To the nearest x	and y satisfy the equation y satisfy the equ	on $2x^2 + 2y^2 - 10x + 14$ ne value of $2025x^y$?	y + 37 = 0.	() 407 41		
(a) 81.97	(b) 122.39	(c) 182.74	(d) 272.85	(e) 407.41		
13. A list is made of random from th	of all positive unreduced is list.	proper fractions with de	nominators less than 11 .	A fraction is chosen at		
What is the probability that the fraction's decimal representation terminates?						
(a) $\frac{16}{37}$	(b) $\frac{8}{15}$	(c) $\frac{5}{9}$	(d) $\frac{5}{8}$	(e) $\frac{80}{99}$		
14. A circle is inscr inscribed in the <i>THOUSANDTH</i> ,	ibed in an equilateral tri- square. Let A be the a what is a/A ?	angle. Then a square is irea of the triangle and	inscribed in the circle. a the area of the smalle	Finally, another circle is r circle. To the nearest		
(a) 0.239	(b) 0.247	(c) 0.302	(d) 0.428	(e) 0.551		
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15. Adelaide invested \$1000 into an investment that earned 10.0% the first year, 15.0% the second year, and 20.0% the third year. Bernard invested \$1000 into an investment that had a constant annual interest rate (with interest paid once per year) over the same three years. At the end of the three years, Adelaide's and Bernard's investments were worth the same amount.

What, to the nearest HUNDREDTH of a percentage point, was the annual interest rate for Bernard's investment?

(a) 14.93% (b) 14.96% (c) 15.00% (d) 15.07% (e) 15.18%

16. Suppose you have determined that 30% of the e-mails you received last week were "spam."

Further, the word *free* occurred in 60% of the spam e-mails from last week but in only 10% of the non-spam emails. Suppose that you choose an e-mail from last week at random. Given that the word *free* appears in the e-mail, compute, to the nearest *HUNDREDTH*, the probability that the e-mail is "spam."

- (a) 0.50 (b) 0.64 (c) 0.72 (d) 0.83 (e) 0.86
- 17. Sixteen students in a dance contest have numbers 1 to 16. When they are paired up, they discover that each couple's numbers add to a perfect square.

What is the largest *difference* between the two numbers for any couple?

(a) 5 (b) 7 (c) 9 (d) 10 (e) 12

18. Solve the equation " $6^{(x+1)} = 2^{(3-2x)}$ " for x, rounding x to the nearest THOUSANDTH. What is the sum of the three digits after the decimal point in that rounded value of x?

- (a) 8 (b) 10 (c) 11 (d) 15 (e) 21
- 19. For what real-number values of a does the equation $x^2 + ax + a = 0$ have two real solutions for x, with one solution being positive and the other being negative?

(a) all a less than 0	(b) all a such that	(c) all a such that	(d) all a greater	(e) There are no such
	0 < a < 4	a < 0 OR a > 4	than 4	real values for a .

20. The vertices of a quadrilateral are the points (0,0), (2,7), (6,5), and (10,0). What is the area of the quadrilateral? (a) 38 (b) 39 (c) 40 (d) 41 (e) 42

- 21. A super-special limited-edition iced coffee drink was stolen from the refrigerator in the faculty lounge. You are the detective called in to investigate. You have determined that the following statements are true.
 - Either Euclid or Germain was in the computer lab at the time of the heist.
 - If Euclid was in the computer lab at the time of the heist, then Apollonius stole the iced coffee drink.
 - If Conon was in the department office at the time of the heist, then Diophantus stole the iced coffee drink.
 - If Germain was in the computer lab at the time of the heist, then Hypatia stole the iced coffee drink.
 - If Conon was not in the department office at the time of the heist, then Germain was not in the computer lab at the time of the heist.

Who stole the iced coffee drink?

(a) Apollonius (b) Diophantus (c) Euclid (d) Germain (e) Hypatia

22. A day is called **minimal** if the number of its month and the number of its day are relatively prime. (That is, they have no common factors larger than 1.) What is the smallest number of minimal days in any month?

(a) 9 (b) 10 (c) 11 (d) 14 (e) 15

23. Let m and l be two lines that are perpendicular to each other and are both tangent to a circle of radius 6. To the nearest HUNDREDTH, find the area bounded by the two lines and the circle.
(a) 7.73 (b) 9.19 (c) 10.93 (d) 12.99 (e) 15.45

24. Suppose that $x \neq y$ and that the two sequences of numbers below are both arithmetic sequences:

$$x, a_1, a_2, a_3, y \text{ and } b_1, x, b_2, b_3, y, b_4.$$

What is the value of $\frac{b_4 - b_3}{a_2 - a_1}$? (a) 2/3 (b) 4/3 (c) 5/3 (d) 7/3 (e) 8/3

25. A sequence is given by the following specifications: We start with a positive first term a_1 , and then for every integer $n \ge 1$, we have $a_{n+1} = \frac{12}{2a_n+5}$.

As n grows (that is, as we look at more and more terms of the sequence), the terms of the sequence most closely approximate which of the following?

(a) $\frac{4}{3}$ (b) $\frac{17}{12}$ (c) $\frac{3}{2}$ (d) $\frac{14}{9}$ (e) $\frac{19}{12}$

- 26. Zenobia works weekdays for \$15 an hour, Saturdays for \$20 an hour, and Sundays for \$25 an hour. Last month, she worked 180 hours total, earning \$3235. How many *more* weekday hours than Sunday hours did she work last month?
 - (a) 63 (b) 65 (c) 68 (d) 70 (e) 73
- 27. Consider the following snippet of pseudocode:

Suppose that the user inputs 5 for the value n. How many stars are printed?

- (a) 15 (b) 23 (c) 25 (d) 29 (e) 35
- 28. In the figure below left, a rectangle of height w is inscribed in a semicircle of diameter 8. Find the area of the rectangle in terms of w.
 - (a) $2\sqrt{2}w$ (b) w^2 (c) $2w^2$ (d) $2w\sqrt{16-w^2}$ (e) $2\sqrt{2}w\sqrt{16-w^2}$

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- 29. In the diagram above right, the quadrilateral ABCD is a square. (The diagram is not to scale.) To the nearest degree, what is x?
 - (a) 42° (b) 46° (c) 49° (d) 51° (e) 53°

30. What are the solutions (for x) of the equation $2x^2 + ix + 3 = 0$? (a) -2i, 3i (b) $-\frac{3}{2}i$, i (c) -i, i (d) $-\frac{1}{2}i$, $\frac{3}{2}i$ (e) 0, 3i

31. Suppose that $4^x - 4^{(x-1)} = 24$. Which of the following is equal to $(2x)^x$? (a) $\sqrt{5}$ (b) $5\sqrt{5}$ (c) 25 (d) $25\sqrt{5}$ (e) 125

32. Let "f(x)" mean " $x^2 - 4x$ " for any real number x. Now let t be a real number. What is f(t-3)? (a) $t^2 - 4t - 12$ (b) $t^2 - 4t - 3$ (c) $t^2 - 6t + 9$ (d) $t^2 - 10t + 9$ (e) $t^2 - 10t + 21$