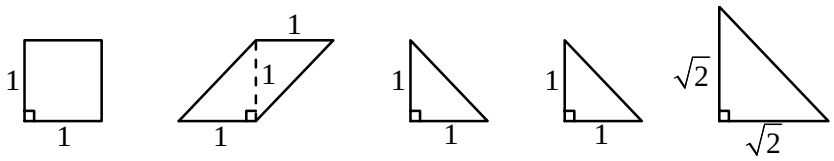
2019 SCSU MATH CONTEST
11th and 12th Grade Test

DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

1. Find the number of solutions to the equation $|-x| = -x$.

   A. zero B. one C. two D. three E. more than three

2. One square can be constructed using all five shapes shown below: one square, one parallelogram and three right triangles with the measurements shown. What is the perimeter of the square that is formed?



   A. $4\sqrt{2}$ B. $4+2\sqrt{2}$ C. $2+4\sqrt{2}$ D. 8 E. $4+4\sqrt{2}$

3. Consider the set of all five-digit integers such that the product of its five digits is 7!
   What is the sum of the five digits in the largest element of the set?

   A. 27 B. 28 C. 29 D. 30 E. 31

4. The number of distinct positive integral divisors of 30^4, excluding 1 and 30^4, is:

   A. 30 B. 110 C. 123 D. 125 E. 243

5. What is the sum of all the solutions to the equation $|\sin x| = \sin x + 2\cos x$ for $0 \le x < 2\pi$?

   A. $\frac{3\pi}{2}$ B. 2π C. $\frac{9\pi}{4}$ D. $\frac{5\pi}{2}$ E. $\frac{9\pi}{2}$

6. If nine pens cost eleven dollars and *x* cents, and thirteen pens cost fifteen dollars and *y* cents, then $x+y$ is:

   A. 70 B. 84 C. 106 D. 107 E. 112

7. Ten identical marbles are randomly distributed to five children.
   It is possible that some children receive no marbles.
   What is the probability, rounded to three decimal places, that each child gets at least one marble?

   A. 0.031 B. 0.084 C. 0.126 D. 0.200 E. 0.500

8. Find the ratio of the area of a rectangle to its perimeter if its length and width are the zeros of the polynomial $p(x) = x^2 - 25x + 10$.

   A. $\frac{1}{5}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{2}{5}$ E. 5

9. The diagonal of square #1 is $a+b$.
   If square #2 has twice the area of square #1, find the perimeter of square #2.

   A. $(a+b)^2$ B. $\sqrt{2}(a+b)^2$ C. $2(a+b)$ D. $\sqrt{8}(a+b)$ E. $4(a+b)$

10. If $\log_{2x} 216 = x$, where x is a positive real number, then x is:

A. A perfect square B. A perfect cube C. A non-integer rational number D. An irrational Number E. An integer larger than 2

11. $@x@y@ = (2x-y)^2$ and $\#a\#b\# = (a+2b)^2$. If $\#5\#(n-1)\# = 5n^2+4n$ and $n<6$, then $@2n@2@$ equals

A. 4 B. 9 C. 16 D. 36 E. 49

12. Flip a fair coin n times.
What is the smallest possible value of n such that the probability of at least two heads exceeds 0.8?

A. 4 B. 5 C. 6 D. 7 E. 8

13. If $x<-2$, then $\left|1-\left|1+x\right|\right|$ equals

A. $2+x$ B. $-2-x$ C. x D. $-x$ E. $2-x$

14. What is the sum of all twenty-four different permutations of the number 1234?

A. 6006 B. 60,000 C. 60,666 D. 66,066 E. 66,660

15. If $f(x) = x^2+bx+c$, $f(1)=9$ and $f(3)-f(2)=8$, find $f(4)$.

A. $\frac{1}{2}$ B. 11 C. 19 D. 23 E. 33

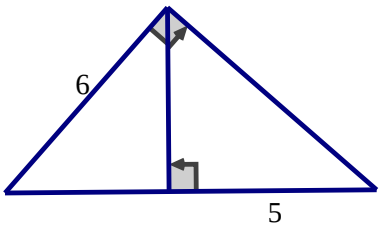
16. The roots of the equation $ax^2+bx+c=0$ will be reciprocals of each other if:

A. $a=b$ B. $a=c$ C. $b=c$ D. $a=bc$ E. $c=ab$

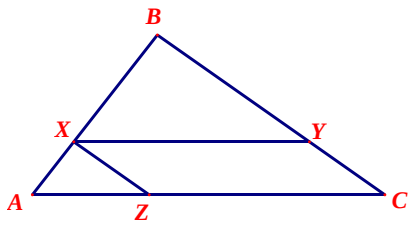
17. How many positive integer divisors of 49,000,000 are not perfect squares?

A. 115 B. 117 C. 147 D. 7,000 E. 16,800,000

Use the diagrams below to answer questions 18 and 19.



QUESTION 18



QUESTION 19

18. What is the area, in square units, of the large right triangle with lengths and right angles as shown?

A. $\frac{39}{2}$ B. 18 C. $9\sqrt{5}$ D. $15\sqrt{2}$ E. $18\sqrt{5}$

19. In the diagram, $\overline{XY}$ is parallel to $\overline{AC}$, and $\overline{XZ}$ is parallel to $\overline{BC}$. Triangle ABC has an area of 98 square units, and trapezoid $XYCA$ has an area of 48 square units. What is the area, in square units, of trapezoid $BCZX$?

A. 50 B. 56 C. 70 D. 76 E. 90

20. Given a three-digit number, make a four-digit number by putting a 7 as the rightmost digit. Then add 48 to the resulting four-digit number. Then remove the rightmost digit to obtain a three-digit number.
If the result is 395, what is the sum of the digits of the original three-digit number?

A. 12 B. 14 C. 17 D. 19 E. 22

21. Twelve nonnegative integers have a mean of 2019.
Which one of the following is the largest possible value of the median for the numbers?

A. 2019 B. 3460 C. 3461 D. 4037 E. 4038

22. What is the probability of rolling a number less than 3 at least 3 times in 5 rolls of a six-sided die?

A. $\frac{2}{15}$ B. $\frac{17}{81}$ C. $\frac{1}{3}$ D. $\frac{2}{5}$ E. $\frac{15}{32}$

23. Ten children in a classroom line up for lunch. Nora insists on being somewhere ahead of Tim.
If Nora's demand is to be satisfied, in how many ways can the children line up?

A. 90 B. 8! C. 9! D. $\frac{10!}{2}$ E. 10!

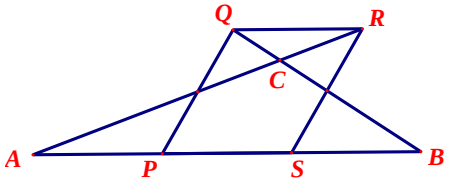
24. The sum of all zeros of $f(x)=x^3-4x^2+x+26$, including complex zeros, is:

A. -2 B. 4 C. 6 D. 8 E. 13

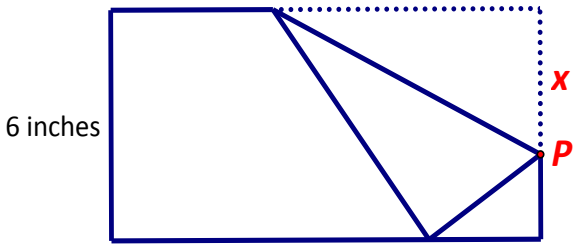
25. A subset of *n* elements is randomly chosen from the set $\{1,2,3,\ldots,9\}$.
What is the smallest *n* that guarantees that the subset must contain two elements whose sum is 10?

A. 5 B. 6 C. 7 D. 8 E. 9

Use the diagrams below to answer questions 26 and 27.



QUESTION 26



QUESTION 27

26. In the diagram above, $\overline{AR}$ bisects $\overline{PQ}$, $\overline{BQ}$ bisects $\overline{SR}$, and the area of parallelogram *PQRS* is *k*.
What is the area of triangle *ABC*?

A. k B. $\frac{9k}{8}$ C. $\frac{6k}{5}$ D. $\frac{5k}{4}$ E. $2k$

27. A 12-inch by 6-inch rectangular piece of paper is positioned so the 12-inch side is at the bottom. A point *P* is selected on the right edge. The paper is folded at *P* so the upper right corner ends up on the opposite 12-inch side. Let *x* be the distance from the top right corner to point *P*. Assume $x>3$.
An expression for the length of the crease is:

A. $\frac{x^{3/2}}{\sqrt{x-3}}$ B. $\frac{3x}{\sqrt{x-9}}$ C. $\frac{x^{3/2}}{\sqrt{2x+9}}$ D. $\frac{x}{\sqrt{x^2-36}}$ E. $\frac{6-x}{\sqrt{x^2-36}}$

28. In a geometric sequence of positive terms, the difference between the fifth and fourth terms is 576, and the difference between the second and first terms is 9. What is the sum of the first five terms of this sequence?

A. 768 B. 1023 C. 1024 D. 1061 E. 4095

29. A three-digit integer N is chosen at random. The probability that $\log_2 N$ is an integer is:

A. $\frac{1}{300}$ B. $\frac{3}{899}$ C. $\frac{1}{100}$ D. $\frac{9}{899}$ E. $\frac{1}{450}$

30. Ten people play a round-robin chess tournament (every person plays everyone else exactly once). In each game, the winner gets 3 points, the loser gets 0 points, and in the case of a draw (tie), each of the players gets 1 point. The total number of points awarded in the tournament is 131. How many games are draws?

A. 1 B. 2 C. 3 D. 4 E. 5

31. Suppose $f(x+1,y)=f(x,y)+y+1$, $f(x,0)=x$ and $f(x,y)=f(y,x)$ for all real numbers x and y.
The value of $f(10,3)$ is:

A. 29 B. 35 C. 39 D. 43 E. 49

32. Let $F(1)=-1$ and $F(n)=F(n-1)+\frac{1}{2}$ for all integers $n>1$. Find $F(101)$.

A. 49 B. 50 C. 51 D. 52 E. 53

33. How many distinct polygons can be drawn with all vertices taken from the 12 hour points on a clock?

A. 12 B. 144 C. 4017 D. 4083 E. 4096

34. Suppose a and b are positive real numbers such that $a+b=ab=a^2-b^2$. Find $a-b\sqrt{5}$.

A. -2 B. -1 C. 1 D. 2 E. 5

35. In how many three-digit numbers are at least two of the digits the same?

A. 18 B. 100 C. 164 D. 200 E. 252

36. In triangle ABC, $AB=16, AC=15,$ and $BC=9$. What is the ratio $\frac{B}{A}$?
That is, find the ratio of the measure of angle B and the measure of angle A.

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. $\frac{3}{4}$ D. 2 E. 3

37. A line segment is divided so that the ratio of lesser part to the greater part is the same as the ratio of the greater part to the whole.

If R is the ratio of the lesser part to the greater part, then the value of $R^{(R^2+R^{-1})}+R^{-1}$ is:

A. 2 B. $2R$ C. R^{-1} D. $2+R$ E. $2+R^{-1}$