2018 SCSU MATH CONTEST
11th and 12th Grade Test

DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic graphing calculators are not allowed.

1. Find the remainder when 4^{2018} is divided by 7.
   A. 0 B. 1 C. 2 D. 3 E. 4

2. A chord of length 2 in the circle $x^2+y^2-2x-2y=-1$ has one endpoint at $(2,1)$. Find the coordinates of the other endpoint.
   A. $(0,1)$ B. $\left(1,-\sqrt{3}\right)$ C. $(2,3)$ D. $(4,-1)$ E. $\left(4,\sqrt{2}\right)$

3. The variable *n* represents the smallest positive integer such that $\sqrt{1050n}$ is a whole number. Find the sum of the digits of *n*.
   A. 6 B. 8 C. 12 D. 13 E. 15

4. A math class has 28 students, and the ratio of boys to girls is 3:4. If the number of boys remains the same, how many girls must join the class to make the ratio of boys to girls 3:5?
   A. 2 B. 4 C. 7 D. 10 E. 12

5. The sequence $6, a, b, c, 14406, \ldots,$ is geometric. Find the mean of $a, b,$ and c.
   A. 798 B. 2891 C. 4800 D. 5187 E. 7206

6. The height of a stone thrown from a cliff is given by $h(t)=-16t^2+68t+50,$ where height *h* is measured in feet and time *t* in seconds. Find the average velocity of the stone from $t=2$ to $t=2.5,$ in feet per second.
   A. -4 B. -2 C. 0 D. 2 E. 4

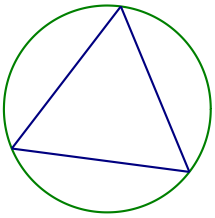
7. A regular octagon is formed by cutting congruent isosceles right triangles from each corner of a square. If the side length of the square is one unit, then the legs of each isosceles triangle are how many units long?
   A. $\frac{2-\sqrt{2}}{3}$ B. $\frac{2-\sqrt{2}}{2}$ C. $\frac{1+\sqrt{2}}{3}$ D. $\frac{1+\sqrt{2}}{2}$ E. $\frac{2+\sqrt{2}}{3}$

8. The sum of the digits of a seven-digit positive integer is 3. How many such positive integers exist?
   A. 15 B. 16 C. 22 D. 28 E. 33

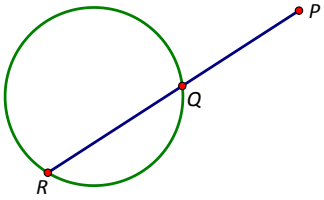
9. A point is randomly selected from within a triangle having vertices $(-3,-7), (-3,8),$ and $(12,12)$. Find the probability that the *y*-coordinate of the point is greater than the *x*-coordinate.
   A. $\frac{3}{4}$ B. $\frac{7}{8}$ C. $\frac{4}{9}$ D. $\frac{7}{12}$ E. $\frac{11}{15}$

10. Find the value of the following sum: $\cos 0^\circ+\cos 1^\circ+\cos 2^\circ+\cos 3^\circ+\cdots+\cos 360^\circ$.
    A. 0 B. 1 C. $\frac{\pi}{2}$ D. π E. 2π

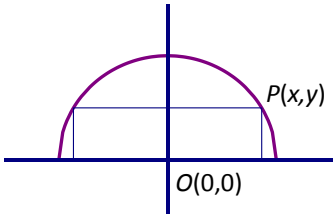
Use the diagrams below to answer questions 11, 12 and 13.



Question 11



Question 12



Question 13

11. An equilateral triangle with sides of length 3 units is inscribed in a circle. Find the area of the region that is inside the circle, but outside the triangle, in square units.

A. $\frac{6\pi-9\sqrt{3}}{2}$ B. $9\pi-\frac{9\sqrt{3}}{8}$ C. $\frac{12\pi-9\sqrt{3}}{4}$ D. $\frac{9\pi-27}{4}$ E. $3\pi-\frac{27}{8}$

12. Point *P* is outside a circle and 13 inches from the center of the circle. Line segment *PR* intersects the circle at points *Q* and *R*. Segment *PQ* is 9 inches and segment *QR* is 7 inches. Find the radius of the circle.

A. 3 inches B. 4 inches C. 5 inches D. 6 inches E. 7 inches

13. A rectangle is inscribed in a semi-circle centered at the origin with a radius of 2 units. Find the area of the rectangle in terms of *x*, the *x*-coordinate of point *P*.

A. $x\sqrt{4-x^2}$ B. $x\sqrt{x^2-4}$ C. $4x^2$ D. $2x\sqrt{4-x^2}$ E. $4x\sqrt{x^2-4}$

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14. Find the number of real solutions to the equation $|x|^2+|x|-6=0$.

A. Zero B. One C. Two D. Three E. Four

15. If $\log 2=a$ and $\log 3=b$ then $\log_5 12$ equals:

A. $\frac{2a+b}{1-a}$ B. $\frac{a+2b}{1-a}$ C. $\frac{a+b}{1+a}$ D. $\frac{2a+b}{1+a}$ E. $\frac{a+2b}{1+a}$

16. Find the equation of the slant asymptote of the graph of the function $f(x)=\frac{x^2+5x+4}{x+3}$.

A. $y=0.5x+2$ B. $y=0.5x+1.8$ C. $y=x+1.8$ D. $y=x+2$ E. $y=1.5x+2.8$

17. The mass of a radioactive substance decays from 842 grams to 571 grams in 23.2 years. Find the approximate half-life of this radioactive substance, in years.

A. 13.0 B. 26.1 C. 36.5 D. 40.1 E. 41.4

18. Suppose $f(x)=x^4-4x^2+k$. For what values of *k* will the graph of $f(x)$ have four distinct *x*-intercepts?

A. $k<4$ B. $0<k<4$ C. $k>-4$ D. $-4<k<4$ E. $|k|>4$

19. Find an algebraic expression equivalent to $\sin\left(\tan^{-1}x\right)$.

A. $\frac{1-x^2}{x}$ B. $\sqrt{1+x^2}$ C. $\frac{1}{1+x^2}$ D. $\frac{x}{\sqrt{1+x^2}}$ E. $\frac{\sqrt{1-x^2}}{x}$

20. If Q quarts of motor oil cost a total of C cents, how many gallons of this oil can you buy for D dollars?

A. $\frac{DQ}{25C}$ B. $\frac{4DQ}{C}$ C. $\frac{DQ}{400C}$ D. $\frac{DQ}{4C}$ E. $\frac{25DQ}{C}$

21. A regular hexagon is inscribed in a circle. The area of the hexagon is approximately what percent of the area of the circle?

A. 78.3 B. 80.1 C. 82.7 D. 83.3 E. 85.7

22. Find the sum of the arithmetic series $3+7+11+\ldots+1283$.

A. 206,403 B. 207,046 C. 412,806 D. 414,092 E. 415,378

23. Consider the three expressions $2^{10^{x+1}}$, $3^{2\left(10^x\right)}$ and 10^{10^x} for $x \geq 0$. Order the expressions from smallest to largest.

A. $2^{10^{x+1}}, 3^{2\left(10^x\right)}, 10^{10^x}$ B. $3^{2\left(10^x\right)}, 10^{10^x}, 2^{10^{x+1}}$ C. $10^{10^x}, 2^{10^{x+1}}, 3^{2\left(10^x\right)}$

D. $3^{2\left(10^x\right)}, 2^{10^{x+1}}, 10^{10^x}$ E. $2^{10^{x+1}}, 10^{10^x}, 3^{2\left(10^x\right)}$

24. Suppose that the nth term of a sequence is given by $P_n = 3.2P_{n-1}\left(1-P_{n-1}\right)$. If $P_0 = 0.4$, find the approximate value of P_{1000}.

A. 0 B. 0.513 C. 0.522 D. 0.768 E. 0.799

25. A pile of nickels, dimes, and quarters has twice as many quarters as nickels and twice as many dimes as quarters. The value of these coins is less than \$20. What is the maximum number of coins that could be in the pile?

A. 126 B. 147 C. 183 D. 205 E. 239

26. We are given that k is a real number and that the sum of $3k$ and $\frac{10}{k}$ is the cube of k. Find the value of the square of k.

A. -7 B. -3 C. 2 D. 5 E. 13

27. A cube has a surface area of 64 square units. Find the surface area of the largest sphere that fits inside the largest cylinder that fits inside the cube, in square units.

A. $\frac{32\pi}{3}$ B. 16π C. $\frac{64\pi}{9}$ D. 64π E. $\frac{128\pi}{3}$

28. Find the value of y when solving the system of real-valued equations:

$$\begin{aligned} x\sqrt{5} + y\sqrt{7} &= 2 \\ x\sqrt{7} - y\sqrt{5} &= 6 \end{aligned}$$

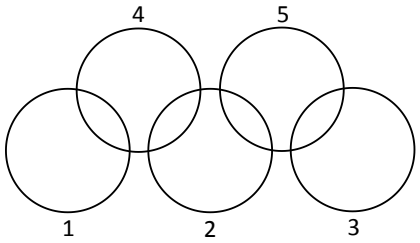
A. $\frac{\sqrt{5}}{6}+\frac{\sqrt{7}}{2}$ B. $3\sqrt{7}-\sqrt{5}$ C. $\sqrt{7}+3\sqrt{5}$ D. $3\sqrt{5}-\sqrt{7}$ E. $\frac{\sqrt{7}}{6}-\frac{\sqrt{5}}{2}$

29. What is the measure of the acute angle formed by the hour and minute hands of a clock that reads 2:15?

A. 5° B. 7.5° C. 22.5° D. 28° E. 30°

Use the diagrams below to answer questions 30 and 31.

Question 30



Question 31

30. The length of sides *AD* and *BC* of parallelogram *ABCD* are increased by 35 percent. The height of *ABCD* is decreased by 35 percent. Find the percent change in the area of the parallelogram.

A. 0% B. +9.75% C. −12.25% D. +19% E. −25.5%

31. The five numbered rings represent five different colors. The colors are Red, Green, Blue, Yellow and Orange, but not necessarily in that order. It is known that

- the Blue ring does not intersect the Green ring,
- the Yellow ring intersects only the Blue ring, and
- the Red ring is to the right of the Green ring and on the same level as the Green ring.

Which ring number is colored Orange?

A. 1 B. 2 C. 3 D. 4 E. 5

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32. A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers choose their spaces at random from among the available spaces. Andy then arrives in his truck, which requires 2 adjacent spaces. Find the probability that Andy is able to park his truck in this parking lot.

A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{15}{32}$ D. $\frac{3}{4}$ E. $\frac{17}{28}$

33. Find the sum of the series $\frac{1}{\log_2(2018!)}+\frac{1}{\log_3(2018!)}+\frac{1}{\log_4(2018!)}+\cdots+\frac{1}{\log_{2018}(2018!)}$.

A. $\frac{1}{2018}$ B. $\frac{1}{2}$ C. 1 D. 2 E. 2018

34. On a trick six-sided die, the probability of rolling a specific value is given in the table at the right. The trick die and a standard fair die are each rolled once. Find the probability that the sum of the roll values is a multiple of 3.

TRICK SIX-SIDED DIE						
Roll Value	1	2	3	4	5	6
Probability	$\frac{5}{24}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{24}$	$\frac{1}{6}$

A. $\frac{1}{3}$ B. $\frac{5}{12}$ C. $\frac{19}{72}$ D. $\frac{11}{36}$ E. $\frac{35}{144}$

35. Ayden, Ellery, Leyton, Bennett, and Catherine are all different ages. Exactly one of the following five statements is true.

- Ayden is the oldest.
- Bennett is older than Leyton.
- Catherine is older than Ellery.
- Leyton is not the youngest.
- Ellery is not the oldest.

Who is the oldest?

A. Ayden B. Ellery C. Leyton D. Bennett E. Catherine