2016 SCSU MATH CONTEST
11^{th} and 12^{th} Grade Test

DIRECTIONS: Select the BEST response from those given. Scientific and graphing calculators are allowed. Symbolic and programmable calculators are not allowed.

1. Four distinct integers are represented by w, x, y, and z. We are given that $w > x$ and that z is the maximum of the four numbers. Which one of the following additional pieces of information, if true, would allow us to place the integers in order from smallest to largest?

A. $w > y$ B. $x > y$ C. $y > x$ D. $z > x$ E. $z > w$

2. Suppose that x is 5% larger than y and y is 10% larger than z. By what percent is z *smaller than* x? Round your answer to the nearest *TENTH* of a percentage point.

A. 9.5% B. 12.4% C. 13.4% D. 15.0% E. 15.5%

3. Five members of a wrestling team are weighed one at a time. The average weight of the first two wrestlers is 3 more pounds than the weight of the first wrestler. The average weight of all the wrestlers weighed increases by 3 pounds each time another wrestler is weighed. How much heavier (in pounds) is the last wrestler than the first?

A. 12 B. 15 C. 24 D. 30 E. Impossible to tell

4. Suppose that $a, b,$ and c are consecutive *odd* integers. What is the value of $a^2 - 2b^2 + c^2$?

A. 2 B. $2b$ C. 4 D. $4b$ E. 8

5. What is the units digit of 7^{2016}?

A. 1 B. 3 C. 5 D. 7 E. 9

6. When data are skewed left, the mean will *usually* be . . .

A. . . . greater than the median. B. . . . negative. C. . . . equal to the median.
D. . . . positive. E. . . . less than the median.

7. A ladder leans against a wall. The top of the ladder is 8 feet above the floor. If you slide the bottom of the ladder 2 feet *farther* from the wall, the top of the ladder slides to the base of the wall. How many feet long is the ladder?

A. 10 B. 13 C. 15 D. 17 E. 19

8. Find the area of the polygon in the plane with vertices at the points $(-1,1)$, $(1,4)$, $(2,5)$, $(5,2)$, and $(4,0)$.

A. 15.5 B. 16.0 C. 16.5 D. 17.0 E. 17.5

9. Consider these statements:

I. A normal density curve is symmetric.
II. A normal density curve has a peak at the mean.
III. The area under a normal density curve is 1.

Which of the statement(s) above is/are *true*?

A. II only B. I and II only C. I and III only D. II and III only E. I, II, and III

10. Which of the following is true of the graph of $f(x) = -a\sin(\pi - bx)$ for any positive a and b?

A. The amplitude is a, the period is π, and the x-intercepts are spaced at $\frac{\pi}{a}$-unit intervals.

B. The amplitude is a, the period is $\frac{\pi}{b}$, and the x-intercepts are spaced at $\frac{\pi}{2b}$-unit intervals.

C. The amplitude is a, the period is $\frac{2\pi}{b}$, and the x-intercepts are spaced at $\frac{\pi}{b}$-unit intervals.

D. The amplitude is $-a$, the period is π, and the x-intercepts are spaced at $\frac{\pi}{b}$-unit intervals.

E. The amplitude is $-a$, the period is $\frac{\pi}{b}$, and the x-intercepts are spaced at $\frac{\pi}{2b}$-unit intervals.

11. For two bases a and 7, we have $421_a = 1323_7$. Find a.
A. 8 B. 9 C. 10 D. 11 E. 12

12. Find the algebraic expression for $\sin(\tan^{-1} x)$.
A. $\frac{1+x^2}{x}$ B. $\sqrt{1+x^2}$ C. $\frac{1}{1+x^2}$ D. $\frac{x}{\sqrt{1+x^2}}$ E. $\frac{\sqrt{1-x^2}}{x}$

13. Solve for x: $\log_2 x + \log_2(x-2) = 3$.
A. $x = 4$ B. $x = -2$ or 4 C. $x = 3$ D. $x = 1$ or 3 E. $x = \frac{5}{2}$

14. The state legislature will select a six-member committee from among seven liberals, nine moderates, and six conservatives. In how many ways can the committee be chosen so that it contains exactly three liberals?
A. 1134 B. 2268 C. 15,925 D. 90,720 E. 573,300

15. Suppose $f(x) = x^4 - 4x^2 + c$. What condition(s) guarantee that the graph of f has four x-intercepts?
A. $0 < c < 4$ B. $c < 4$ C. $|c| < 4$ D. $-4 < c < 0$ E. $c < -4$ or $c > 4$

16. One-half of the water is poured out of a full container. Then one-third of the remaining water is poured out. The process continues with one-fourth for the third pouring, one-fifth for the fourth pouring, and so on. After how many pourings does exactly one-tenth of the original water remain?
A. 3 B. 5 C. 8 D. 9 E. 10

17. What is the median of the patterned list $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \cdots, 100, \cdots, 100$?
A. 68 B. 69 C. 70 D. 71 E. 72

18. Suppose that $f(n)$ is a function on the positive integers.
If $f(1) = 0$ and for every $n \geq 2$, $f(n) = 2n + f(n-1)$, what is $f(6)$?
A. 10 B. 16 C. 28 D. 40 E. 42

19. In 23.2 years, 842 grams of a radioactive substance decays exponentially to 571 grams of that substance. To the nearest *TENTH* of a year, what is the half-life of this substance?
A. 13.0 B. 16.0 C. 36.0 D. 40.1 E. 41.4

20. Alice must pick a team of four people to work on a project, out of a pool of ten employees, but she knows that Bob and Charlie don't work well together. How many different teams are possible that do not include Bob and Charlie *together*?
A. 126 B. 154 C. 182 D. 210 E. 238

21. A function has the following properties: $x = a$ is a vertical asymptote; $(0, 3)$ is a y-intercept; and $y = b$ is a horizontal asymptote. Which of the functions below satisfies these properties?
A. $j(x) = \frac{bx+3a}{x+a}$ B. $k(x) = \frac{3x-a}{x-b}$ C. $l(x) = \frac{x-3b}{x-a}$ D. $m(x) = \frac{bx-3a}{x-b}$ E. $n(x) = \frac{bx-3a}{x-a}$

22. Seven objects, labeled A, B, C, D, E, F, and G, are to be displayed in a row. How many different orderings of the objects are possible in which object G is to the left of object C?
A. 1260 B. 2520 C. 2660 D. 2820 E. 5040

23. The angle of elevation to the top of a water tower from point *A* on the ground is 22.3°. From point *B*, which is 100 feet closer to the tower, the angle of elevation is 27.2°. What is the height of the tower to the nearest foot?
A. 27 B. 51 C. 203 D. 223 E. 395

24. A parabolic arch has a height of 16 inches and a span of 40 inches. Determine the height, in inches, of the arch at a point 5 inches from the center.
A. 1 B. 14 C. 15 D. 15.5 E. 16

25. We are given that $3 + 2i$ is a zero of $f(x) = x^2 + bx + c$, where b and c are real numbers. Find $b + c$.
A. 0 B. 5 C. 7 D. 10 E. 19

Use these figures to answer questions 26 and 27. Do not assume that the figures are constructed to scale.

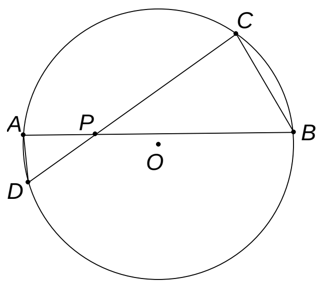

Figure 1

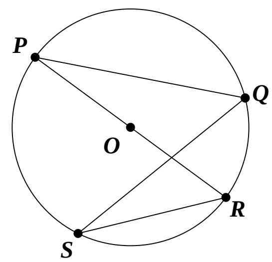

Figure 2

26. Two chords intersect within a circle as shown in Figure 1 (above left). The area of $\triangle PAD$ is 8, and AP:PC =2:5. Find the area of $\triangle PCB$.
A. 20 B. 30 C. 32 D. 40 E. 50

27. In Figure 2 (above right), $m\angle PRS = 50°$ and $SR = \frac{2}{3}PQ$ Find the measure of $\angle QPR$.
A. 30° B. 40° C. 45° D. 50° E. 60°

28. Consider the following pseudocode:

```
input(n)
s := 0
for i = 3 to n
     for j = 2 to i
     s := s + j
     next j
next i
output(s)
```

Suppose the input is $n = 5$. What is the output?
A. 15 B. 22 C. 28 D. 37 E. 42

29. Find the sum of the *real* solutions to the equation $9^x + 98 = 7(3^{x+1})$.
A. $log_3 14$ B. $log_3 21$ C. $log_3 98$ D. 14 E. 21

30. Determine the number of real solutions (x, y) to the system of equations given below.

$$2|x| - x + y = 5$$
$$2x + 2y + |y| = 12$$

A. no solutions B. one solution C. two solutions D. three solutions E. four solutions

31. On the island of Smullyania, every inhabitant is either a *day-knight* (who speaks only the truth during daylight hours and speaks only lies during the non-daylight hours) or a *night-knight* (who does the opposite). On a vacation to Smullyania, you meet three inhabitants: Arvid, Beatrice, and Cadwallader.

Arvid says, “Beatrice is the only night-knight here.”

Beatrice flashes him an irritated look and says, “I’m the *only* one of the three of us who *isn’t* a night-knight.”

You look to Cadwallader for clarification. He shrugs and says, “It’s nighttime now, and I speak lies at night.”

Which of the following is true? (You may assume that the sun neither rose nor set during the conversation!)

A. Arvid is a night-knight; Beatrice is a day-knight; it is impossible to tell what Cadwallader is.
B. Arvid is a night-knight; Beatrice is a day-knight; Cadwallader is a day-knight.
C. Arvid is a night-knight; it is impossible to tell what Beatrice is; Cadwallader is a night-knight.
D. Arvid is a day-knight; Beatrice is a day-knight; Cadwallader is a day-knight.
E. Arvid is a day-knight; Beatrice is a night-knight; Cadwallader is a night-knight.

32. Abel randomly selects *two* distinct integers from the set { 1, 2, 3, 4 }. Independently, Borel randomly selects *one* integer from the set { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 }. What is the probability that the *sum* of the two integers Abel selected is *greater than* Borel’s integer?

A. 0.2 B. 0.3 C. 0.4 D. 0.5 E. 0.6

33. Each face of a cube is assigned a different positive integer. Then each vertex is assigned the sum of the integer values on the three faces that meet at the vertex. Finally, the vertex numbers are added. What is the largest number that *must* be a divisor of the final sum for *every* possible numbering of the faces?

A. 1 B. 2 C. 3 D. 4 E. 6

Use these figures to answer questions 34 and 35. Do not assume that the figures are constructed to scale.

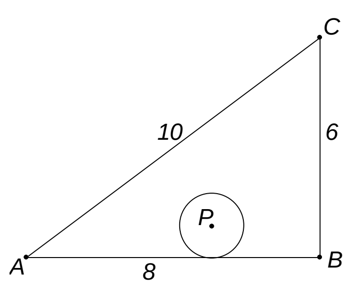

Figure 3

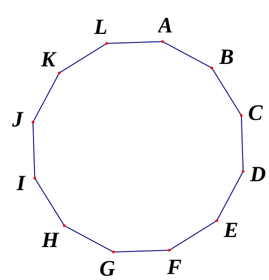

Figure 4

34. The side lengths of $\triangle ABC$ are 6, 8, and 10, as shown in Figure 3 (above left). A circle with center P and radius 1 rolls around inside $\triangle ABC$, always remaining tangent to at least one side, and returns to its original position. What distance has P traveled?

A. 10 B. 12 C. 14 D. 15 E. 18

35. The polygon $ABCDEFGHIJKL$ shown in Figure 4 (above right) is a *regular* 12-gon (“dodecagon”). Its second-shortest diagonal measures 10 units. Find the area of polygon BAD.

A. $\frac{25}{2}(\sqrt{3}-1)$ B. $\frac{5}{2}\sqrt{3}+6$ C. $12-\frac{5}{2}\sqrt{3}$ D. $3\sqrt{3}+\frac{5}{2}$ E. $2\left(\sqrt{3}+\frac{5}{2}\right)$