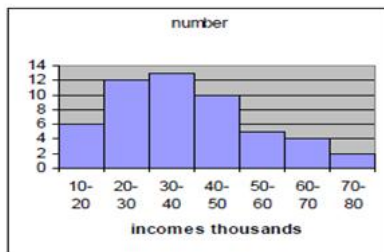


**2014 SCSU MATH CONTEST**  
**11<sup>th</sup> and 12<sup>th</sup> GRADE**

**DIRECTIONS:** Select the **BEST** completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

- Laurel can paint a room in 5 hours. Hardy can paint the same room in 3 hours. Ramanujan can paint the same room in 3.5 hours. If they all work together (and can stay out of each other's way), about how many *minutes* will it take them to paint that room? Round your answer to the nearest *minute*.  
 (a) 72                      (b) 73                      (c) 74                      (d) 75                      (e) 76
- In triangle  $\triangle XYZ$ ,  $\angle XYZ$  is a right angle,  $\angle XZY$  measures 39 degrees, and  $XZ$  is 17 cm. Find  $YZ$  (to the nearest hundredth of a centimeter).  
 (a) 9.84                      (b) 10.70                      (c) 11.63                      (d) 13.21                      (e) 13.77
- Kelly invests \$1000 in an account earning a 6% annual interest rate, compounded monthly. Pat invests the same amount at a rate of 6.1%, compounded annually. At the end of ten years, who will have more money—and how much more?  
 (a) Kelly has \$1.00 more than Pat.  
 (b) Kelly has \$11.59 more than Pat.  
 (c) Pat has \$6.78 more than Kelly.  
 (d) Pat has \$2.11 more than Kelly.  
 (e) They have exactly the same amount.
- How many different five-digit integers can be formed with the digits in 11235 (that is, two 1's, a 2, a 3, and a 5)?  
 (a) 12                      (b) 24                      (c) 36                      (d) 60                      (e) 120
- What is the units digit in the base 10 representation of the integer  $43^{2014}$ ?  
 (a) 1                      (b) 3                      (c) 5                      (d) 7                      (e) 9
- The histogram shows the incomes in thousands of dollars of 52 professionals in a certain field. Within what interval does the third quartile ( $Q_3$ ) fall?



- (a) \$30,000–\$40,000      (b) \$40,000–\$50,000      (c) \$50,000–\$60,000      (d) \$60,000–\$70,000      (e) \$70,000–\$80,000
- Suppose that  $x = A$  is the positive solution to the equation  $x^2 = x + 1$ . Simplify the following.

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{A}}}}$$

- (a)  $A$   
 (b)  $\frac{A}{A+1}$   
 (c)  $A^2 + 1$   
 (d)  $\frac{A^2 + 1}{A}$   
 (e)  $A^5 - A^4 + A^3 - A^2 + A - 1$
- The Math Club at Ortho Central High School is made up of 8 girls and 6 boys. From that group, a delegation of 2 girls and 2 boys must be chosen. What is total number of possible delegations that can be chosen from this group?  
 (a) 12                      (b) 48                      (c) 192                      (d) 420                      (e) 1680

9. Divide  $3x^4 - 4x^3 + 5x^2 - 6x + 7$  by  $x - 1$ . What is the remainder?  
 (a)  $-8$  (b)  $-1$  (c)  $0$  (d)  $3$  (e)  $5$
10. Professor Smith gave an exam that had a mean of 50 and a standard deviation of 10. The professor “curved” the exam by adding 10 points to everyone’s score. What are the mean and the standard deviation, respectively, of the altered scores?  
 (a) 50 and 10 (b) 50 and 20 (c) 60 and 10 (d) 60 and 20 (e) It cannot be determined from the given information.
11. Suppose that  $A$  and  $B$  are points on a circle of radius  $r$  with center  $O$ , and that the distance between  $A$  and  $B$  is  $r$ . What is the radian measure of the central angle  $\angle AOB$ ?  
 (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi}{4}$  (c)  $1$  (d)  $\frac{\pi}{3}$  (e)  $\frac{\pi}{2}$
12. High school students in the Cauchy-Riemann school district may take classes in German, French, both, or neither. The district statistician reports that 34% of the students are taking French (and perhaps German as well); 41% of them are taking German (and perhaps French as well); and 38% of them are taking neither.  
 Suppose you pick a high school student at random and learn that she is taking a French class. What is the probability (to the nearest percentage point) that she is also taking a German class?  
 (a) 9% (b) 13% (c) 24% (d) 38% (e) 61%
13. A regular octagon is inscribed in a circle. Approximately what portion of the area of the circle lies within the octagon?  
 (a) 82% (b) 84% (c) 86% (d) 88% (e) 90%
14. Consider the following pseudocode:

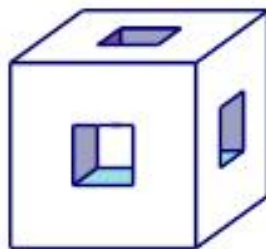
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input(n)
P := 1
for i = 1 to n do
  S := 0
  for j = 1 to i do
    S := S + j
  next j
  P := P · S
next i
output(P)

```

What is the output  $P$  of this pseudocode if the input is  $n = 4$ ?

- (a) 10 (b) 24 (c) 48 (d) 120 (e) 180
15. A fair coin is tossed 6 times. What is the probability of tossing *at least* 4 heads *in a row*?  
 (a)  $\frac{1}{4}$  (b)  $\frac{1}{8}$  (c)  $\frac{1}{16}$  (d)  $\frac{1}{32}$  (e)  $\frac{1}{64}$
16. A cube of edge 3 inches has a square hole, 1 inch by 1 inch, cut from the center of each of its faces straight through to the opposite side. Find the area (in square inches) of the surface of the resulting shape. Note that this includes the area of the parts of the surface that are inside the original cube.



- (a) 64 (b) 68 (c) 72 (d) 78 (e) 96
17. Divide, and write your answer in the form  $a + bi$  (where  $i = \sqrt{-1}$ ):  $\frac{6 - 5i}{3 + i}$   
 (a)  $\frac{13}{10} - \frac{21}{10}i$  (b)  $-\frac{13}{10} + \frac{21}{10}i$  (c)  $\frac{23}{10} - \frac{9}{10}i$  (d)  $\frac{23}{10} + \frac{9}{10}i$  (e)  $2 - 5i$

18. Finish the sentence: *The variance of a data set can never be...*

- (a) ... zero.
- (b) ... negative.
- (c) ... larger than the standard deviation.
- (d) ... smaller than the standard deviation.
- (e) We cannot assert anything about the variance without seeing the data.

19. A radioactive substance decays from 330.0 grams initially to 250.8 grams in 11.1 years. To the nearest tenth of a year, what is the half-life of the substance?

- (a) 27.4                      (b) 27.6                      (c) 27.8                      (d) 28.0                      (e) 28.2

20. If one root of the quadratic equation  $x^2 + bx + c = 0$  is  $x = 1 + 4i$  (where  $b$  and  $c$  are real), then what is the value of  $c$ ?

- (a) 1                      (b) 5                      (c) 9                      (d) 13                      (e) 17

21. Joe wishes to create a rectangular pen for his pigs by using 84 yards of fencing. One side will be the wall of a barn and will remain unfenced. What is the maximum area (in square yards) that can be enclosed?

- (a) 441                      (b) 710                      (c) 784                      (d) 833                      (e) 882

22. The equation  $2014^{(x^2)} = 2015^{(2x+1)}$  has two solutions. To the nearest hundredth, what is their *sum*?

- (a) -1.62                      (b) 0.00                      (c) 1.27                      (d) 2.00                      (e) 3.15

23. What is the domain of the function  $f$  given by

$$f(x) = \sqrt{x - \sqrt{2 - x}} \text{ ?}$$

- (a)  $[-2, 1]$                       (b)  $[-2, 2]$                       (c)  $[0, 2]$                       (d)  $[1, 2]$                       (e)  $[2, \infty)$

24. Suppose that  $\sin \theta = x$  and  $\theta$  is in Quadrant II. Find  $\sec \theta$ .

- (a)  $\frac{1}{\sqrt{1-x^2}}$                       (b)  $-\frac{1}{\sqrt{1-x^2}}$                       (c)  $\frac{x}{\sqrt{1-x^2}}$                       (d)  $-\frac{x}{\sqrt{1-x^2}}$                       (e)  $\frac{\sqrt{1-x^2}}{x}$

25. Suppose we define a Boolean function " $\clubsuit$ " by the following truth table:

$x$	$y$	$x \clubsuit y$
1	1	1
1	0	1
0	1	0
0	0	1

Which of the following is *NOT* logically equivalent to  $x \clubsuit y$ ?

- (a)  $x \text{ OR } \text{NOT}(y)$
- (b)  $\text{NOT}(\text{NOT}(x) \text{ AND } y)$
- (c)  $(x \text{ AND } y) \text{ OR } (\text{NOT}(y))$
- (d)  $x \text{ OR } (\text{NOT}(x) \text{ AND } \text{NOT}(y))$
- (e)  $(\text{NOT}(x) \text{ OR } y) \text{ AND } \text{NOT}(x \text{ AND } y)$

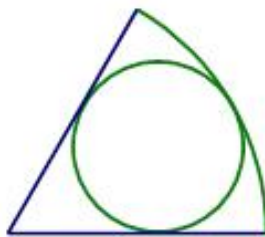
26. Excluding itself and 1, how many positive integer divisors does  $360^4$  have?

- (a) 123                      (b) 238                      (c) 364                      (d) 478                      (e) 583

27. Triangle  $\triangle PQR$  is a 3-4-5 right triangle with its right angle at  $P$ . The points  $X$  on  $\overline{PQ}$ ,  $Y$  on  $\overline{QR}$ , and  $Z$  on  $\overline{PR}$  are chosen so that  $\square PXYZ$  is a square. What is the area of that square?

- (a)  $\frac{3}{4}$                       (b)  $\frac{4}{3}$                       (c)  $\frac{25}{16}$                       (d)  $\frac{25}{9}$                       (e)  $\frac{144}{49}$

28. The first three terms of a certain geometric sequence are  $a - 1$ ,  $a$ , and  $a + 4$ . What is the sum of the *first four* terms?  
 (a)  $\frac{31}{2}$  (b) 17 (c)  $\frac{85}{3}$  (d)  $\frac{85}{2}$  (e) 85
29. The equation  $\tan(3x) + \sec(3x) = 2$ ,  $x \in [0, 2\pi)$ , with  $x$  measured in radians, has more than one solution. To the nearest thousandth, what is the *sum* of the solutions of this equation?  
 (a) 2.094 (b) 6.927 (c) 8.166 (d) 9.425 (e) 18.850
30. In isosceles triangle  $\triangle ABC$ ,  $AB = BC$ , and  $M$  is the midpoint of segment  $\overline{AC}$ . Also,  $M$  is 5 inches from segment  $\overline{BC}$  and 13 inches from  $C$ . What is  $BM$  (to the nearest hundredth of an inch)?  
 (a) 5.20 (b) 5.42 (c) 5.65 (d) 5.89 (e) 6.14
31. Every inhabitant of the Republic of Smullyania is one of two types: either a Truthteller (who *always* tells the truth) or a Liar (who *always* lies). You meet three inhabitants of the island: Adelaide, Bernard, and Cornelius.  
 Adelaide says, "I'm a Truthteller."  
 Bernard says, "I'm the only Liar in our group."  
 And Cordelia adds, "Bernard is right—he is the only Liar in our group."  
 Which of the following is correct?  
 (a) Adelaide and Bernard are Liars; Cordelia is a Truthteller.  
 (b) Adelaide and Cordelia are Truthtellers; Bernard is a Liar.  
 (c) Adelaide is a Truthteller and Bernard is a Liar; it is impossible to tell what type Cordelia is.  
 (d) Bernard and Cordelia are Liars; it is impossible to tell what type Adelaide is.  
 (e) All three are Liars.
32. The line  $l$  is given by the equation  $y = 2x + 1$ . Point  $P$  has coordinates  $(0, 5)$ . Point  $Q$  is the reflection of  $P$  across the line  $l$ . What is the *sum* of the coordinates of point  $Q$ ?  
 (a)  $\frac{29}{5}$  (b) 6 (c)  $\frac{31}{5}$  (d)  $\frac{32}{5}$  (e)  $\frac{33}{5}$
33. A circle is inscribed in a sector that comprises  $\frac{1}{6}$  of a larger circle. The inscribed circle is tangent to the two radii *and* the arc of the larger circle. To the nearest hundredth, what is the ratio of the sector's area to the inscribed circle's area?



- (a) 1.33 (b) 1.41 (c) 1.50 (d) 1.73 (e) 2.00
34. Today's date is 04/03/14 ("14" represents the year 2014). In how many ways can these six digits (0, 4, 0, 3, 1, and 4) be rearranged to form a valid date in the same format (mm/dd/yy)?  
 (a) 21 (b) 26 (c) 28 (d) 30 (e) 31
35. A square with sides of length  $x$  has vertices  $P$ ,  $Q$ ,  $R$ , and  $S$  (listed in clockwise order). Let  $M$  be the midpoint of segment  $\overline{PQ}$  and  $N$  the midpoint of segment  $\overline{RS}$ . Let  $V$  be the intersection of  $\overline{SM}$  and  $\overline{PN}$ , and let  $W$  be the intersection of  $\overline{RM}$  and  $\overline{QN}$ . What is the area of the quadrilateral  $MWNV$ ?  
 (a)  $\frac{x^2}{12}$  (b)  $\frac{x^2}{6}$  (c)  $\frac{x^2}{4}$  (d)  $\frac{x^2}{3}$  (e)  $\frac{x^2}{2}$