2012 SCSU MATH CONTEST
11^{th} and 12^{th} GRADE

DIRECTIONS: Select the <u>BEST</u> completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. Fred can paint a room in 4 hours. Barney can paint the same room in just 3 hours. Working together, how long (to the nearest minute) will the two men take to paint the room?

   (a) 1hr., 43 min. (b) 1hr., 45 min. (c) 1hr., 47 min. (d) 1hr., 49 min. (e) 1hr., 51 min.

2. A stick 14 inches long falls into a round pipe with radius 3 inches. The stick lands with one end in a corner of the closed bottom of the pipe. How far up the wall of the pipe (in inches) is the stick's other end?

   (a) $2\sqrt{5}$ (b) $4\sqrt{10}$ (c) $6\sqrt{10}$ (d) $8\sqrt{10}$ (e) $9\sqrt{10}$

3. Suppose that N is an integer and N^5 ends with the digit 7. What is the last digit of N itself?

   (a) 1 (b) 3 (c) 5 (d) 7 (e) 9

4. You roll a pair of fair six-sided dice one time. What is the probability that the sum of the two numbers is *less than* 6? (*Round to FOUR decimal places.*)

   (a) 0.1389 (b) 0.1667 (c) 0.1944 (d) 0.2778 (e) 0.3636

5. Find the output when the following piece of code is executed.

```
k = 71;
if(k > 90)
    Print ‘‘Awesome!’’;
else
    if (k > 80)
        Print ‘‘Bravo!’’;
    else
        if(k > 70)
            Print ‘‘Come on!’’;
        else
            print ‘‘Ditto’’;
    print ‘‘Good luck’’;
```

   (a) `Awesome!` `Good luck` (b) `Bravo!` `Good luck` (c) `Come on!` `Good luck` (d) `Come on!` `Ditto!` `Good luck` (e) `Ditto!` `Good luck`

6. A commuter airplane flies from Computerville to Statsburg at 160 MPH. On the return trip, with a tailwind, the plane travels at 240 MPH. What is its average speed for the round trip?

   (a) 180 MPH (b) 188 MPH (c) 192 MPH (d) 200 MPH (e) 204 MPH

7. Find the number of distinguishable permutations of the letters in `SAINTCLOUDSTATE`.

   (a) $\dfrac{15!}{11! \cdot 4!}$ (b) $\dfrac{15!}{2! \cdot 2! \cdot 3!}$ (c) $15!$ (d) 15^{11} (e) 11^{15}

8. In my notebook I've drawn two triangles, $\triangle ABC$ and $\triangle DEF$. Segment AB is congruent to segment DE, and segment BC is congruent to segment EF. Also, $\angle BCA$ is congruent to $\angle EFD$. Which theorem guarantees that $\triangle ABC$ and $\triangle DEF$ are congruent triangles?

   (a) "Side-Angle-Side" (b) "Side-Side-Angle" (c) "Side-Side-Side" (d) the "Pythagorean" Theorem (e) There is no theorem that guarantees this.

9. Two non-zero real numbers x and y satisfy the equation $xy = x - y$. Which of the following is a possible value of

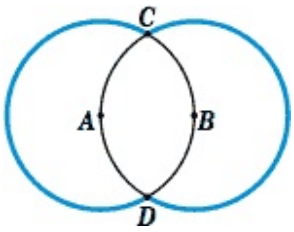
$$\frac{x}{y} + \frac{y}{x} - xy?$$

   (a) -2 (b) $-\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{2}$ (e) 2

10. How many terminating zeros are there in $25!$?

(a) 4 (b) 6 (c) 8 (d) 10 (e) 12

11. Find the length of the boundary of the intersection of the two circles. Points A and B are the centers of the circles, and $AB = 12$ cm.



(a) 40 cm (b) 16π cm (c) 54.5 cm (d) 27π cm (e) 145.5π cm

12. Suppose a bag contains 5 apples and 3 oranges. You draw one piece of fruit at a time from this bag, replacing each piece of fruit after drawing it. What is the probability that your third draw is the second time you draw an apple?

(a) 0.1465 (b) 0.1786 (c) 0.2930 (d) 0.3572 (e) 0.4286

13. Which of the following statements is (are) true?

$$\begin{aligned}
&\text{I.} && (\ln x)^2 = 2\ln x \\
&\text{II.} && \log_4\left(3x^4\right) = 4\log_4(3x) \\
&\text{III.} && \log(x-y) = \frac{\log x}{\log y} \\
&\text{IV.} && \log_3\left(81^5 \cdot 3^{x-20}\right) = x \\
&\text{V.} && \log_5\left(\tfrac{125}{6-y}\right) = 3 + \log_5(y-6)
\end{aligned}$$

(a) I and V only (b) II and III only (c) III and IV only (d) III and V only (e) IV only

14. Suppose that the function f is given by $f(x) = \sin(kx)$, where k is some positive integer. How many values of x between 0 and 2π satisfy the equation

$$|f(x)| = \frac{1}{2}?$$

(a) k (b) $2k$ (c) $4k$ (d) $8k$ (e) $16k$

15. Find the output when the following piece of code is executed.

```
n = 10;
i = 1;
sum = 0;
while( i < n )
{
    sum = sum + i;
    i = i + 1;
}
print sum;
```

(a) 36 (b) 45 (c) 55 (d) 74 (e) 100

16. Let $x^2 + bx + c$ be a quadratic polynomial with real coefficients. Suppose that one root of this polynomial is $2 - i$. What is the value of the constant term c?

(a) -4 (b) -3 (c) 2 (d) 5 (e) There is not enough information to determine c.

17. There are nine numbers in a data set: { 1, 2, 3, 4, 5, 6, 7, 8, 9 }. In how many different ways can we choose three numbers (without replacement), so that their median is greater than 5? (Recall that the median is the number in the middle.)

(a) 30 (b) 34 (c) 39 (d) 45 (e) 52

18. Every inhabitant of the island of Smullyania is one of two types: either a Truthteller (who *always* tells the truth) or a Liar (who *always* lies). You meet three inhabitants of the island: Adelaide, Bernard, and Cornelius.

Adelaide says, "Bernard and Cornelius are both Truthtellers."

Bernard adds, "Adelaide and Cornelius are of different types."

Which of the following is correct?

(a) Both Adelaide and Bernard are Truthtellers.
(b) Both Adelaide and Cornelius are Liars.
(c) Both Bernard and Cornelius are Truthtellers.
(d) Bernard and Cornelius are different types.
(e) Exactly one of the three is a Liar, but it is impossible to determine who.

19. Suppose that $\sin x + \cos x = -\frac{1}{5}$ and $\frac{3}{4}\pi \le x \le \pi$. Find the value of $\cos(2x)$.

(a) $-\frac{24}{25}$ (b) $-\frac{7}{25}$ (c) $\frac{7}{25}$ (d) $\frac{7}{24}$ (e) $\frac{24}{25}$

20. Suppose that $f(x) = \sqrt{x-3}$ and $g(x) = 2x - 3$. Find $\left(f \circ g^{-1}\right)(7)$.

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) $\sqrt{5}$ (e) $\sqrt{6}$

21. Which of the following numbers is (are) rational?

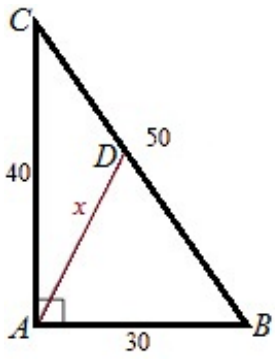
I. 2.012 2012 2012 2012 . . .
II. 0.1 01 001 0001 00001 . . .
III. 3.141592654
IV. $(343)^{-\frac{2}{3}}$
V. $\sin\left(\frac{25\pi}{6}\right)$

(a) I, II, and III only (b) I, III, and IV only (c) I, III, IV, and V only (d) II and IV only (e) II, IV, and V only

22. Dumbert plans to buy lottery tickets until he wins for the first time. If his probability of winning is 0.01 on each ticket, find the probability that he will need to buy at least 100 tickets. (*Round to the nearest HUNDREDTH.*)

(a) 0.01 (b) 0.37 (c) 0.50 (d) 0.73 (e) 0.99

23. In the diagram below, $\triangle ABC$ is a right triangle, and the segment AD divides it into two triangles of equal perimeters.



What is the length of segment AD?

(a) 12 (b) $6\sqrt{5}$ (c) 24 (d) $12\sqrt{5}$ (e) 30

24. The base-four representation of a number x is given by

$$(12133221231231212312212 2)_{\text{four}}.$$

What is the first digit (the leftmost digit) of x in base sixteen? (Recall that in base sixteen, the numbers 10 to 15 are represented by A through F, respectively.)

(a) 4 (b) 6 (c) 9 (d) A (e) C

25. Evaluate $i + 2i^2 + 3i^3 + 4i^4 + \cdots + 60i^{60}$, where $i = \sqrt{-1}$.

(a) 20 (b) 30 (c) $60i$ (d) $30 - 30i$ (e) $60 + 60i$

26. The diagonals of a rhombus are 20 and 12. Determine the radius of a circle inscribed in the rhombus.

(a) $\frac{15}{\sqrt{34}}$ (b) $\frac{18}{\sqrt{34}}$ (c) $\frac{30}{\sqrt{34}}$ (d) $2\sqrt{34}$ (e) $12\sqrt{34}$

27. Five cards are arranged on a table. Each card has a *letter* on one side and a *number* on the other.

You are told the following:

Every card that has a Z on one side has an EVEN *number on the other side.*

What is the minimum number of cards you must turn over in order to verify that this claim is true?

(a) one (b) two (c) three (d) four (e) five

28. The three sides of a triangle measure 29 cm, 32 cm, and 35 cm. To the nearest degree, what is the measure of the smallest *angle* of the triangle?

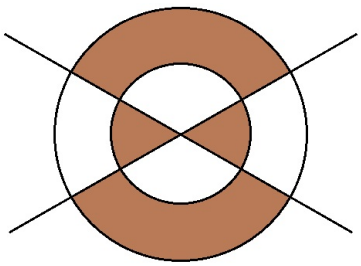
(a) 45° (b) 47° (c) 49° (d) 51° (e) 53°

29. How many integer pairs (x, y) satisfy the equation $(x - 2)(x - 10) = 3^y$?

(a) one (b) two (c) three (d) four (e) five

30. Two distinct lines pass through the center of two concentric circles of radii 2 and 1. The area of the shaded region in the diagram is $\frac{7}{5}$ of the area of the unshaded region. (The diagram is not to scale.)

What is the radian measure of the acute angle formed by the two lines?



(a) $\frac{\pi}{4}$ (b) $\frac{7\pi}{24}$ (c) $\frac{\pi}{3}$ (d) $\frac{3\pi}{8}$ (e) $\frac{5\pi}{12}$

31. A fair coin is flipped 5 times. If the second one is a "Head" (H), what is the probability that there is exactly one pair of consecutive "Heads" (HH) in the sequence?

(a) $\frac{1}{16}$ (b) $\frac{3}{32}$ (c) $\frac{1}{8}$ (d) $\frac{3}{16}$ (e) $\frac{1}{4}$

32. Let the function f be given by

$$f(x) = \frac{\sqrt{(2x+5)(x-4)}}{x^2 - 9}.$$

How many of these intervals are subsets of the domain of f?

$$(-\infty, -\pi), \quad \left(-\pi, -\frac{5}{2}\right), \quad \left(-\frac{5}{2}, \pi\right), \quad \left(\pi, \frac{3\pi}{2}\right), \quad (2\pi, \infty)$$

(a) one (b) two (c) three (d) four (e) five