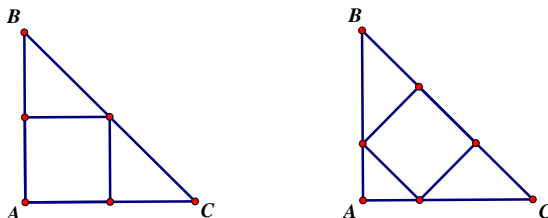


2010 SCSU MATH CONTEST
11th and 12th Grade Test

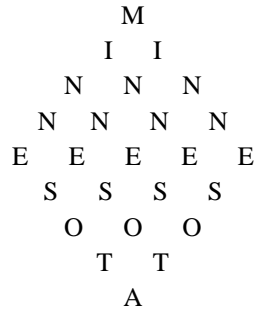
DIRECTIONS: Select the BEST completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

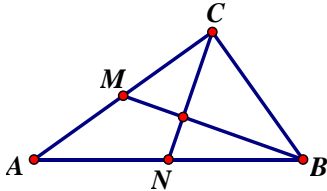
- If $y = (\log_2 3)(\log_3 4) \cdots (\log_{31} 32)$, then
 - $2 < y < 4$
 - $y = 4$
 - $4 < y < 5$
 - $y = 5$
 - $5 < y < 6$
- In the figures below, $\triangle ABC$ is an isosceles right triangle. In the figure to the left, the area of the inscribed square is 225 cm^2 . What is the area (in cm^2) of the square inscribed in the same $\triangle ABC$ as shown in the figure to the right?



- 180
 - 200
 - 210
 - 225
 - 240
- The largest of nine numbers is 89. The median of the nine numbers is 70 and the mean (or average) of them is 72. If we change the largest one to be 98, what are the new median (m) and mean (\bar{x})?
 - $m = 79, \bar{x} = 81$
 - $m = 70, \bar{x} = 81$
 - $m = 70, \bar{x} = 73$
 - $m = 71, \bar{x} = 73$
 - $m = 71, \bar{x} = 72$
 - Students need to take a placement test in order to register for a math class. They can take the test at most three times. If they fail the first time, they can take it again, and if they fail the second time, they can take the exam once more. The passing rates for the three tries are 40%, 30% and 30%, respectively. What is the overall passing rate for the test?
 - 69.4%
 - 70%
 - 70.6%
 - 79%
 - 100%
 - Consider the equation $\sin(x) = mx$ for some real number m . Which of the following could *not* represent the number of real solutions to this equation?
 - 1
 - 2
 - 3
 - 5
 - 2009
 - Two ferry boats sail back and forth across a river, each traveling at a constant speed, and turning back without any loss of time. They leave opposite shores at the same instant, pass for the first time 900 feet from one shore, continue on their way to the banks, return and pass for the second time 200 feet from the opposite shore. How many feet wide is the river?
 - 1350
 - 2200
 - 2500
 - 2900
 - 2700
 - Let f be a one-to-one function whose inverse function is given by the formula $f^{-1}(x) = 2x^3 + 3x + 1$. What is the value of $(f \circ f)(6)$? ($f \circ f$ indicates the composition of f with itself.)
 - 0
 - 1
 - 451
 - 2,334,888
 - None of these
 - If $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x , then $\sum_{n=1}^{2010} \lfloor \log_2 n \rfloor =$
 - 18000
 - 18064
 - 18434
 - 29291
 - $\lfloor \log_2(2010!) \rfloor$
 - The product of all real roots of the equation $x^{\log_{10} x} = 10$ is
 - 1
 - $\frac{1}{10}$
 - 1
 - 10
 - 100

10. How many ways are there to spell MINNESOTA by starting at the top of the diamond and moving downward, one row at a time, to adjacent letters? (One of the possible paths is shown.)



- a. 10 b. 64 c. 70 d. 126 e. 128
11. In this figure, $\triangle ABC$ is a right triangle, and \overline{BM} and \overline{CN} are perpendicular medians. If $BC = 1$, then $BM =$
- a. $\frac{\sqrt{5}}{2}$ b. $\frac{\sqrt{6}}{2}$ c. $\sqrt{2}$
d. $\frac{3\sqrt{2}}{2}$ e. $2\sqrt{2}$
- 
12. A game consists of tossing a fair coin until you see two consecutive heads (HH). Then you win. Given the information that Jim won the game at the 5th toss, what is the probability that his result was HTTHH?
- a. 0.00032 b. 0.008 c. 0.125 d. 0.333 e. 0.5
13. Three numbers are chosen at random without replacement from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the probability that their median is 7?
- a. 0.15 b. 0.175 c. 0.23 d. 0.33 e. 0.70
14. If $\sum_{n=0}^{\infty} \cos^{2n} \theta = 4$, then $\cos 2\theta =$
- a. $-\frac{\sqrt{3}}{2}$ b. $-\frac{1}{2}$ c. 0 d. $\frac{1}{2}$ e. $\frac{\sqrt{3}}{2}$
15. If $(x, y, z)^* = \frac{xy + yz + zx}{x^2 + y^2 + z^2}$, evaluate $(3, 2, -4)^*$.
- a. $\frac{-14}{29}$ b. $\frac{14}{29}$ c. $\frac{23}{29}$ d. $\frac{44}{29}$ e. None of these
16. If w is 10% larger than x , x is 20% larger than y , and y is 25% smaller than z , by what percentage is w smaller than z ?
- a. 1% b. 1.5% c. 15% d. 17% e. 18%
17. Define $a \ominus b$ to be:
a, if $a = b$
 $a \ominus (b - a)$, if $a < b$
 $b \ominus (a - b)$, if $a > b$
What is the value of $36 \ominus 200$?
- a. 4 b. 24 c. 36 d. 48 e. 64
18. Let $ABCD$ be a square and Q a circle that circumscribes the square. What is the probability that a random point in Q falls within the square?
- a. $1/\pi$ b. $1/2$ c. $2/\pi$ d. $2/\pi^2$ e. $3/\pi^2$

19. The coefficient of y^{-6} in $(y^{-3} - 2y^2)^7$ is
 a. -280 b. -70 c. -7 d. 35 e. 140
20. Excluding 1 and 1,944,810,000, how many positive integer divisors does $210^4 = 1,944,810,000$ have?
 a. 208 b. 254 c. 256 d. 623 e. 625
21. In a TV survey, it is found that 35 people watch comedies, 20 watch detective shows and 17 watch sports. Of these, 7 watch both comedies and detective shows, 4 watch both sports and detective shows, 6 watch comedies and sports and only 2 watch all three types. How many viewers responded?
 a. 52 b. 55 c. 57 d. 58 e. 61
22. Suppose $f(x) + (1+x)f(-x) = x + 2010$. Then $f(2010) + f(-2010) =$
 a. 0 b. 1 c. 2 d. 2009 e. 2011
23. A function which has the same graph as that of $y = x$ is
 a. $y = \sqrt{x^2}$ b. $y = \frac{x^2}{x}$ c. $y = a^{\log_a x}$, where $a > 0, a \neq 1$
 d. $y = \log_a a^x$, where $a > 0, a \neq 1$ e. none of these
24. If $(1-2x)^7 = a_0 + a_1x + a_2x^2 + \dots + a_7x^7$, then the value of $a_0 + a_1 + a_2 + \dots + a_7$ is
 a. -2 b. -1 c. 0 d. 1 e. 2
25. If $|\cos \theta| = 1/5$, $5\pi/2 < \theta < 3\pi$, then the value of $\sin(\theta/2)$ is
 a. $-\frac{\sqrt{10}}{5}$ b. $\frac{\sqrt{10}}{5}$ c. $-\frac{\sqrt{15}}{5}$ d. $\frac{\sqrt{15}}{5}$ e. none of these
26. The line symmetric to the line $2x + 3y - 6 = 0$ with respect to the point $(1, -1)$ is
 a. $3x - 2y + 2 = 0$ b. $2x + 3y + 7 = 0$ c. $3x - 2y - 12 = 0$ d. $2x + 3y - 8 = 0$ e. $2x + 3y + 8 = 0$
27. The angles of a pentagon are in arithmetic progression. One of the angles, in degrees, must be:
 a. 45 b. 54 c. 90 d. 108 e. 135
28. The number of solutions (x, y) to the equation $2^{2x} - 3^{2y} = 55$ in which x, y are integers is
 a. 0 b. 1 c. 2 d. 3 e. more than 3
29. If the parabola $y = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$ and $(2, -3)$, then the value of $a + b + c$ is:
 a. -4 b. -2 c. 0 d. 1 e. 2
30. If $x_k = x_{k-1} + \frac{1}{2}$ for $k = 1, 2, \dots, n$ and $x_0 = 1$, the value of $x_1 + x_2 + \dots + x_n$ is:
 a. $\frac{4n-1}{2}$ b. $\frac{2n+1}{2}$ c. $\frac{n^2+n-6}{2}$ d. $\frac{n^2+3n-4}{4}$ e. $\frac{n^2+5n}{4}$
31. For what real values of C does $x = C^2(x-1)(x-2)$ have real roots?
 a. none b. $-2 < C < 1$ c. $-2\sqrt{2} < C < 2\sqrt{2}$ d. $C > 1$ or $C < -2$ e. all
32. Points P and Q are both in the line segment \overline{AB} and on the same side of its midpoint. P divides \overline{AB} in a ratio of 2:3, and Q divides \overline{AB} in a ratio 3:4. If $PQ = 12$, then the length of \overline{AB} is:
 a. 60 b. 70 c. 130 d. 420 e. 4200