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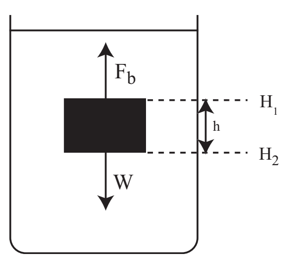
PHYS 231 Lab Exercise: Archimedes' Principle

<u>Objective:</u> You are probably familiar from swimming in pools and/or lakes that when you are partially or fully submerged in water there is an upward force acting on you called a buoyant force. Can you imagine how difficult it would be to tread water if there weren't a large bouyant force pushing upward on you (imagine trying to tread "air" and you get the idea)? In fact, the whole watercraft industry relies on buoyant forces to keep things like boats, canoes and jet-skis all afloat.

This buoyant force can be quantified using **Archimedes' principle**, which states that ***the upward force on a partially or completely submerged object is equal to the weight of the fluid that it displaces.***

In this exercise you will be studying Archimedes' principle, specifically exploiting it measure the specific gravity of a few different solids and a liquid.

<u>Introduction:</u> When an object is submerged into a fluid, it experiences an upward force that is called the buoyant force (or buoyancy). This force is a result of the difference in pressure of the fluid on the top and on the bottom of the submerged object. To show this, we assume for simplicity that the object is a solid cylinder whose cross-sectional area is A and height is h. Then the buoyant force F_b equals the difference in the force of pressure on the top of the cylinder versus the bottom of the cylinder



$$
\begin{aligned}
F_b &= (P_{top} - P_{bottom})A \\
&= [P_{atm} + \rho g H_2 - (P_{atm} + \rho g H_1)]A \\
&= \rho g(H_2 - H_1)A \\
&= \rho g V
\end{aligned}
$$

Here ρ is the density of the fluid and V is the volume of the submerged object that is equal to the volume of displaced fluid. This equation is just another version of the statement of ***Archimedes' Principle*** noted above that ***the upward force on a partially or completely submerged object is equal to the weight of the fluid that it displaces.***

Notice that this statement should match your experiences in the "real world." For example, if an object is less dense than the fluid it is immersed in (*e.g.* – such as wood placed in water), then its weight will be less than the weight of the water it displaces when submerged, and thus the net force on the object will be upward… that is the object will float. However, if an object is instead denser than the fluid it is immersed in (*e.g.* – steel pellets placed in water), then its weight will be greater than the weight of the water displaced and the net force on the object will be downward… that is the object sinks.

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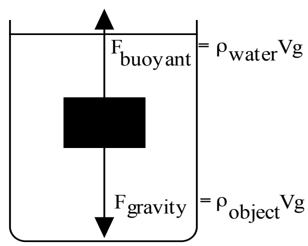
The fact that steel pellets sink in water doesn't mean there is no buoyant force; it simply means the buoyant force was less than the weight of the water displaced. In other words, a submerged object will have an "apparent" weight less than its "true" weight when the buoyant force is not present. In this lab, you will be submerging an object and comparing its mass when not submerged, *m*, to the apparent mass, m_{sub}, when submerged. A force diagram for a submerged object is shown to the right.



Notice that this means the apparent weight of the submerged object is:

$$m_{sub}g = F_{gravity} - F_{buoyant} = m_{object}g - m_{water}g = \rho_{object}Vg - \rho_{water}Vg \quad [1]$$

and the weight of the object outside of water is:

$$mg = F_{gravity} = m_{object}g = \rho_{object}Vg \quad [2]$$

Based on the above equations, it can be shown that the ***specific gravity*** of the object (the ratio of its density versus that of water, $\rho_{water} = 1000\ kg/m^3$) is:

$$\frac{\rho_{object}}{\rho_{water}} = \frac{m}{m - m_{sub}} \quad [3]$$

This method will work if the object is denser than water (and thus sinks) and you will use it to measure the specific gravities of two metal cylinders. However, in addition to this, you will also explore ways of using Archimedes' Principle to measure the specific gravities of solids less dense than water (and thus float) and of liquids (which would mix with water).

<u>Apparatus:</u> Balance, two solid metal cylinders, piece of wood or cork, unknown liquid.

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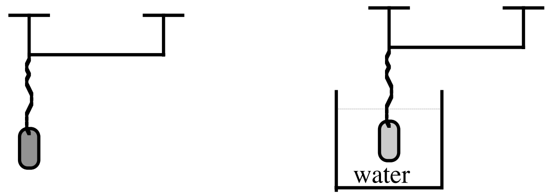
Procedure: **Part I: Specific Gravity of Objects Denser than Water**

a) Hang the smaller of the two metal objects from the hook on the under-side of the balance and determine its mass. ***Record the mass of the small metal object in air, m.***



b) Next arrange to have the object hang completely submerged in water. There is a platform directly below the balance to facilitate this arrangement. ***Measure and record the balance reading for the submerged mass, m_{sub}.***

c) Compute the specific gravity of the small metal object using equation 3. ***Record the specific gravity of the small metal object and use this to compute the density of the small metal object.***

d) Compare your previous determined density for the small metal object to a direct computation of the density:

$$\rho_{metal} = \frac{m}{V}$$

Start by determining the volume of the metal cylinder

   a. Begin by filling the graduated cylinder with water from the sink and ***recording the volume of water in the cylinder (including units)***.
   b. Then carefully place the smaller of the metal objects in the cylinder and ***record the volume of the water plus the metal object (including units)***.
   c. Use this to determine the volume of the small metal object, since it is equal to the volume of the water displaced by that metal object (assuming it is total submerged). ***Record the volume of the small metal object.***

Now ***compute and record the density of the small metal object*** and compare this to your density estimate obtained using Archimedes' principle.

e) Next, repeat the above procedure for the larger metal object. ***Record all your measurements and both computations of the density of the object.***

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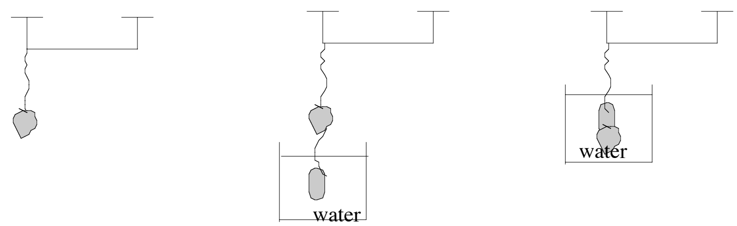
Part II: Specific Gravity of Objects Less Dense than Water

Wood has a density lower than water (which is why it floats). In order to measure the specific gravity of wood, we will have to forcibly sink the wood by tying a denser mass to it (similar to the old movies where gangsters would give their victims "concrete boots" so they would sink).



If we do this, we can compute the specific gravity of wood by using the relationship

$$\frac{\rho_{wood}}{\rho_{water}} = \frac{m}{m_1 - m_2} \qquad [4]$$

where m is the mass of the wooden block, m_1 is the apparent mass of the wood plus a piece of metal when the metal is submerged, and m_2 is the apparent mass of the wood and metal when both of submerged. If you are concerned that equation 4 seems to have popped out of thin air, don't worry; you will get a chance to derive this equation in the conclusion to this lab.

a) Hang the wooden block from the clip on the underside of the balance and ***record its mass, m.***

b) Continue by hanging the heavier of the metal objects to the wooden block and arranging to have only the metal object completely submerged in water. ***Record the apparent mass of the two objects when the metal weight is submerged, call it m_1.***

c) Now arrange to have both the metal weight and the wood completely submerged. Once again, ***record the apparent mass of both objects when both are submerged and call the mass m_2***

d) Use equation 4 to compute and ***record the specific gravity of the wood. Use this to determine the density of wood and record this as well.***

e) Determine the volume of the wood. **DO NOT** place the wood in the graduated cylinder, as it will swell with water and possibly get jammed inside it. Instead, use the Vernier calipers to measure the diameter and length of the wood cylinder. Use this length and diameter to determine the volume of the wood by assuming it is a perfect cylinder, so that volume, V, can be computed using:

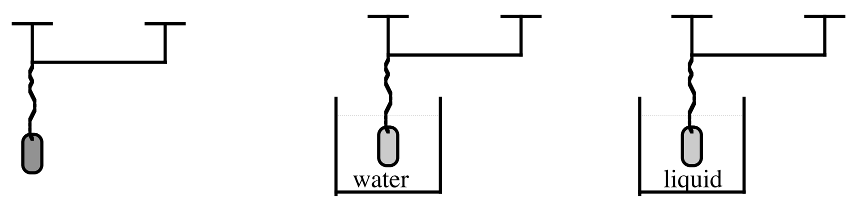
$$V = \pi r^2 l$$

where r is the radius of the cylinder and l is the length. ***Record the length, diameter, radius and the volume you have computed for the wood.***

f) Equipped with direct measurements of the block's mass and of the block's volume, ***compute the wood block's density. Compare this with your estimate computed using Archimedes' principle. Record this work.***

Part III: The Specific Gravities of Liquids

Obviously, we can't measure the density of a liquid by dropping a "block" of the liquid into water, as the two liquids will likely mix. Instead, we can measure the buoyant force of an unknown liquid on a submerged object and compare that to the buoyant force due to water and thus we can compare the density of the unknown fluid to that of water.



We can actually pull this off by simply comparing the apparent mass of an object submerged in water, m_1, versus the apparent mass of the same object submerged in the unknown liquid, m_2, to arrive at the specific gravity of the liquid:

$$\frac{\rho_{liquid}}{\rho_{water}} = \frac{m - m_2}{m - m_1} \qquad [5]$$

where m is the "true" mass of the object. Again, if you are concerned that equation 5 seems to have popped out of thin air, don't worry; you will get a chance to derive this equation in the conclusion to this lab.

a) Take one of the metal objects again, hang it from the underside of the balance and ***record its mass, m.***

b) Then arrange to have the metal object completely submerged in water while you take another reading of the balance, call it m_1. ***Record the submerged mass of the metal object in water, m_1.***

c) The last task is to remove the large beaker with water and submerge the metal object in the unknown fluid contained in the smaller beaker. ***Record the submerged mass of metal object in unknown liquid when the metal object is completely submerged, m_2.***

d) Use equation 5 to compute and ***record the specific gravity of the unknown liquid. Use this to determine the density of the liquid and record this as well.***

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e) The mystery fluid is nothing but salt water. The typical density of seawater is about 1025 kg/m^3.[2] ***Compute the difference between your measurement of the density of your mystery liquid and the reported density of seawater. What might this imply? Record this computation and the results.***

<u>Discussion:</u>

Answer the following questions:

a) Suppose that an ice cube floats in a glass of water filled to the brim. As the ice melts, will the glass overflow?

b) Why do you float more easily in the ocean than in fresh water?

c) Suppose that you have two balloons of the same weight, and they both contain the same amount of helium. One is rigid and the other one is free to expand. When released, which balloon will rise higher?

d) Does the buoyant force on a diving bell deep beneath the ocean's surface have the same value as when the diving bell is just beneath the surface? Explain.

e) A barge containing a tall pile of sand approaches a low bridge and cannot pass under it. Should sand be added to the barge or removed in order to allow it to pass? Explain.

<u>Additional Discussion:</u>

(This section will be assigned at the your instructor's discretion) Now let us return to equations 4 and 5 which appeared too have been pulled out of thin air:

f) The derivation of equation 4 is very similar to the derivation of equation 3, in that the weight of the wood outside the water is:

$$mg = \rho_{wood} V_{wood} g \qquad [6]$$

the apparent weight with just the metal object submerged is

$$m_1 g = \rho_{metal} V_{metal} g + \rho_{wood} V_{wood} g - \rho_{water} V_{metal} g \qquad [7]$$

whereas the apparent weight when both the metal and wood are submerged

[2] Salt crystals have a density of about 2200 kg/m^3, so when we mix salt with water (which has a density of 1000 kg/m^3), the mixture has a higher density than fresh water.

is:

$$m_2 g = \rho_{metal} V_{metal} g + \rho_{wood} V_{wood} g - \rho_{water} (V_{metal} + V_{wood}) g \quad [8]$$

Show that equation 4 is a correct expression by using equations 6, 7, and 8.

g) Similar to the derivation of equation 4, equation 5 can be derived via a combination of force equations for a variety of situations we created to solve for the unknown density. In this case, the weight of the metal outside the water is:

$$mg = \rho_{metal} V_{metal} g \quad [9]$$

the apparent weight of the metal object submerged in water is

$$m_1 g = \rho_{metal} V_{metal} g - \rho_{water} V_{metal} g \quad [10]$$

whereas the apparent weight of the metal object submerged in the mystery liquid is:

$$m_2 g = \rho_{metal} V_{metal} g - \rho_{liquid} V_{metal} g \quad [11]$$

Derive equation 5 using equations 9, 10, and 11.

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DATA SHEET

(You may record the data in an *Excel* spreadsheet if you prefer, but please record the same data)

Part I: Solids more dense than water:

1. Mass of solid in air, m: ______________

   Mass of solid in water, m_1: ______________

   $\frac{\rho_{object}}{\rho_{water}} = \frac{m}{m - m_{sub}}$: ______________ ➔ ρ_{object}: ______________

   Volume of solid: ______________

   ρ_{object} (as directly computed M/V): ______________

2. Mass of solid in air, m: ______________

   Mass of solid in water, m_{sub}: ______________

   $\frac{\rho_{object}}{\rho_{water}} = \frac{m}{m - m_{sub}}$: ______________ ➔ ρ_{object}: ______________

   Volume of solid: ______________

   ρ_{object} (as directly computed M/V): ______________

Part II: Solid less dense than water

Mass of solid in air, m: ______________

Mass of solid in air and sinker in water, m_1: ______________

Mass of solid and sinker in water, m_2: ______________

$\frac{\rho_{wood}}{\rho_{water}} = \frac{m}{m_1 - m_2}$: ______________ ➔ ρ_{object}: ______________

Volume of Wood: ______________

ρ_{object} (as directly computed M/V): ______________

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Part III: Liquid

Mass of solid in air, m: ____________________

Mass of solid in water, m_1: ____________________

Mass of solid in liquid, m_2: ____________________

$\frac{\rho_{liquid}}{\rho_{water}} = \frac{m - m_2}{m - m_1}$: ____________________➔ ρ_{object}: ______________

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