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# PHYS 231 Lab Exercise: Ballistic Pendulum

Objective: Today we will combine two physical concepts (conservation of energy and conservation of momentum) in order to allow you to predict where a projectile launched horizontally from your desktops will land. The key point of this lab is to show you a simple problem, which requires several (simple) physical concepts to solve…how fast is this projectile moving when fired?

Background: Recall that in projectile motion, the horizontal component of the objects velocity is constant while the vertical component is the identical to that of an object in free fall. So if we launch an projectile horizontally then the projectile moves horizontal and vertical distances of

$$\underbrace{x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2}_{x = v_{0x}t} \quad \text{and} \quad \underbrace{y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2}_{y = -\frac{1}{2}gt^2} \qquad \text{(Eqns. 1a \& 1b)}$$

where $g$ = 9.8 m/s$^2$. Rewriting the equation for vertical motion in terms of time, one can combine the above equations into an equation for the range of:

$$x = v_{0x}\sqrt{\frac{2y}{-g}} \qquad \text{(Eqn. 2)}$$

This equation shows that if we measure the distance a projectile drops, $y$ (yes, $y$ will be negative in this case), we can then predict where the projectile will strike the floor below (a horizontal distance $x$ from the starting point) simply by determining the initial horizontal velocity $v_{0x}$. The initial horizontal velocity $v_{0x}$, which we need to predict the location of impact, can be measured using a device called a **ballistic pendulum**.

The ballistic pendulum is capable of launching a projectile horizontally over the edge of the table as well launching the projectile into an inelastic collision with the metal cup on a swinging pendulum. The projectile is fired into the pendulum that is initially hanging at rest. The height to which the combined projectile+pendulum system rises will be a direct function of the projectile velocity which it impacted the pendulum. As outlined in class, from conservation of momentum we know

$$mv_{0x} = (m+M)v_{fx} \qquad \text{(Eqn. 3)}$$

where $m$ is the ball's mass and $M$ is the mass of the pendulum. This still leaves us with two unknowns, the initial and final velocities of the projectile, $v_{0x}$ and $v_{fx}$, just before and just after its "merger" with the pendulum.

To solve for the initial velocity, $v_{0x}$, we need to solve for $v_{fx}$, which we can do using conservation of energy (assuming no frictional forces). If only conservative forces (like gravity) are in play, then we know the total mechanical energy immediately after the collision equals the total mechanical energy when the pendulum+projectile reaches the top of their swing, so:

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$$K_i + U_i = K_f + U_f \qquad \text{(Eqn. 4)}$$

If we define the zero height as the bottom of the pendulum motion, then $U_i = 0$ and therefore:

$$\frac{1}{2}(m+M)v_{fx}^2 = (m+M)g\Delta y \qquad \text{(Eqn. 5)}$$

where $\Delta y$ is the change in height of the center of mass of the combined ball and pendulum. The initial horizontal velocity, $v_{0x}$, for which we are searching can now be written as

$$v_{0x} = \left(\frac{m+M}{m}\right)\sqrt{2g\Delta y} \qquad \text{(Eqn. 6)}$$

The change in height $\Delta y$ and masses $m$ and $M$ are measurable quantities, and therefore $v_{0x}$ can be determined.

**Apparatus:** ballistic pendulum

Procedure: **Part I: Understanding the Derivation of the Previous Equations**

In order to assure that you understand the derivation of the above equations, please answer the following questions. ***You may work together, but any answers must be written down in your own words.***

a) Equation 3 is supposed to be an expression of the conservation of momentum for the ballistic pendulum. Which side of the equation is the initial momentum and which side is the final momentum of the ballistic pendulum? What "event" separates "initial" and "final" here? ***Explain where each of the terms in equation 3 comes from below.***

c) Equation 5 is supposed to be an expression of the conservation of mechanical energy for the ballistic pendulum. ***Explain (or show mathematically) where the two terms come from and write down why we assumed the initial potential energy, $U_i$, and the final kinetic energy, $K_f$, are zero.***

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## Part II: Using the Ballistic Pendulum

And now we ask you to actually place these theoretical derivations regarding ballistic pendulums into practice.

a) Find the mass of the projectile and record it here __________ (units?)

b) Use the ballistic pendulum and allow arm to hang freely without swinging. Fire the ball into the pendulum bob about 4 times recording the rest position of the pendulum on the notched rack each time in a table in *Excel*. Compute the mean and standard deviation of $\Delta y$ and record their values below.

The mean change in height of the ballistic pendulum, $\Delta y$:____________

The standard deviation of $\Delta y$ around the mean value: ____________

c) Finally, using the mean value of $\Delta y$ you computed, calculate your estimate of the initial horizontal velocity of the ball, $v_{0x}$. Show your work and your answer (including units) below.

**Stop!** *Confirm your work with the instructor before continuing…*

## Part III: Predicting the Impact Location and Confirming Prediction

Having a reasonable estimate of the initial horizontal velocity, $v_{0x}$, of the projectile, prepare to launch the ball horizontally over the edge of the table.

a) Carefully measure the vertical displacement, $\Delta y$, which will occur when the ball leaves the launch site and lands on the floor. Record this here.

   The vertical displacement, $\Delta y$, of the ball: ________________

b) Predict where the ball will land on the floor, that is, predict the horizontal displacement, $x$, of the ball during its flight. Record this along with an explanation *in English* of how you derived this estimated landing spot.

c) Tape a piece of white paper on the **floor with a mark indicating where you think the ball will land**. Next place a sheet of carbon paper over the white paper, with a piece of cardboard underneath to protect the floor.

**IMPORTANT:** Before launching the ball, make sure that you have a plywood backdrop placed against the wall for protection.

d) Fire the projectile once and see how close your prediction was. Put the carbon paper back in place, and re-launch the ball about 3 more times. Record these values in a table in *Excel*. Compute the mean difference between your predicted horizontal displacement and the actual value over all four launches?

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# DATA SHEET

**(You may record the data in an *Excel* spreadsheet if you prefer, but please record the same data)**

**Part 1: Projectile fired horizontally**

Vertical fall distance: ___________ Horizontal distance to paper edge: ___________

| ***Trial Number*** | ***Total Distance*** |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| Average | |

Calculated flight time:___________ Initial velocity: ___________

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**Part 2: Projectile fired at an angle**

Angle above horizontal: __________ Horizontal distance to paper edge: __________

| ***Trial Number*** | ***Total Distance*** |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| Average | |

Calculated flight time:__________ Initial velocity: __________