DIRECTIONS: Select the <u>BEST</u> completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. A student spent two-thirds of their money and misplaced one-third of the remainder, leaving them with $18. With how much money did they start?

   (a) $50   (b) $64   (c) $81   (d) $115   (e) $162

2. Solve the following equation for x:
$$\frac{2}{3}(2x - 4) = \frac{1}{2}(x + 3) - 2.$$

   (a) $\frac{5}{13}$ (b) $\frac{11}{13}$ (c) $\frac{13}{11}$ (d) $\frac{13}{5}$ (e) $\frac{23}{5}$

3. At a certain school, 43% of the students play hockey, 47% play soccer, and 29% play neither hockey nor soccer. What percentage of students play both hockey and soccer?

   (a) 15%   (b) 19%   (c) 25%   (d) 28%   (e) 32%

4. I'm thinking of four numbers a, b, c, and d such that
$$a + 1 = b + 2 = c + 3 = d + 4 = a + b + c + d + 5.$$

   What is $a + b + c + d$?

   (a) -5 (b) $-\frac{10}{3}$ (c) $-\frac{7}{3}$ (d) $\frac{5}{3}$ (e) 5

5. Jake bought a toaster at a kitchen supply store that gave a discount of 25% off the original price, P. The total amount Jake paid was t dollars, including a 6% sales tax on the discounted price. Which of the following represents, with coefficient rounded to three decimal places, the original price of the toaster in terms of t?

   (a) $P = 0.810t$ (b) $P = 1.235t$ (c) $P = 1.258t$ (d) $P = 1.291t$ (e) $P = 1.310t$

6. A tank contains 22 liters of an antifreeze-water mix. This mixture is 40% antifreeze. How many liters of this mixture should be *drained* and *replaced* with pure antifreeze so that the mixture will be 50% antifreeze?

   (a) $2\frac{1}{3}$ (b) $2\frac{2}{3}$ (c) $3\frac{1}{5}$ (d) $3\frac{2}{3}$ (e) $4\frac{1}{2}$

7. The equation $x^2 + 14x + y^2 - 8y = 16$ represents a circle. What is the circle's radius?

   (a) 4   (b) 6   (c) 7   (d) 9   (e) 10

8. Three vertices of a parallelogram $PQRS$ are $P(-3, -2)$, $Q(1, -5)$, and $R(9, 1)$, with P and R diagonally opposite each other. What is the *sum* of the coordinates of vertex S?

   (a) 9   (b) 10   (c) 11   (d) 12   (e) 13

9. Find the sum of all positive integers that are divisible by 3 and less than 2024.

   (a) $682,425$ (b) $811,545$ (c) $965,095$ (d) $1,362,825$ (e) $1,364,850$

10. The sum of two numbers, x and y, is 80. When x is divided by 6 and y is divided by 4, neither division has a remainder, and the sum of the quotients is 16. Find the positive *difference* of x and y.

    (a) 4   (b) 8   (c) 12   (d) 16   (e) 20

11. Mr. Earl E. Bird leaves home every day at 8:00 A.M. to go to work. If he drives at an average speed of 40 miles per hour, he will be late by 3 minutes. If he drives at an average speed of 60 miles per hour, he will be early by 3 minutes. How many miles per hour does Mr. Bird need to drive to get to work exactly on time?

    (a) 45   (b) 46   (c) 47   (d) 48   (e) 49

12. Suppose that the two solutions to the quadratic equation $ax^2 - 5x + c = 0$, where a and c are integers, are $x \approx -1.108$ and $x \approx 3.608$. What is the correct value for $a + c$?

    (a) -10 (b) -9 (c) -8 (d) -7 (e) -6

13. What is the sum of all integers that satisfy the inequality $|2x - 4| < 8$?

    (a) 4   (b) 6   (c) 8   (d) 12   (e) 14

14. Suppose that $2^a = 4 \cdot 2^{(b-1)}$ and $2^b = 2^a - 16$. What is $a + b$?

    (a) 6   (b) 7   (c) 8   (d) 9   (e) 10

15. Alvina and Bernadette run on a circular track. The lanes are concentric circles: Alvina's lane is 20 feet farther from the center than Bernadette's lane. Both runners start at the same time, maintain a steady pace of 17 feet per second around the track, and run exactly four laps. To the nearest tenth of a second, how much earlier does Bernadette finish?

    (a) 10.5 seconds   (b) 14.8 seconds   (c) 18.6 seconds   (d) 23.5 seconds   (e) 29.6 seconds

16. Consider the figure below left. Which of the following correctly gives the measure of angle a in terms of angles c and d?

    (a) $c + d - 20°$ (b) $160° - c - d$ (c) $200° - c - d$ (d) $160° - 2c + d$ (e) $200° - c + d$
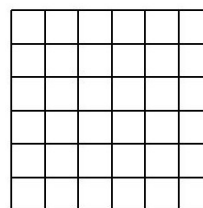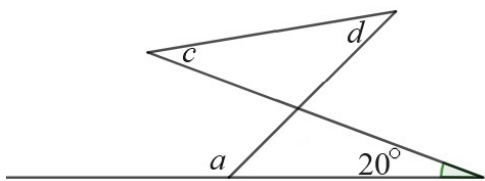
17. Consider a 6-by-6 checkerboard as in the figure above right. How many squares *of any size* are on such a checkerboard?

    (a) 75   (b) 91   (c) 113   (d) 140   (e) 163

18. A deck of cards has 52 cards, including 4 suits of 13 cards each (clubs, diamonds, hearts, spades). Two cards are drawn without replacement. To *FOUR decimal places*, what is the probability that both cards are spades?

    (a) 0.0578   (b) 0.0588   (c) 0.0606   (d) 0.0625   (e) 0.0637

19. Both roots of the quadratic equation $x^2 - 60x + k = 0$ are prime numbers. How many possible values are there for k?

    (a) 0   (b) 1   (c) 2   (d) 4   (e) more than 4

20. In the figure below left, A, B, and D are collinear (and D is between B and A). Suppose that $BD = CD$ and that $\angle B$ measures 35 degrees. (The figure is not to scale.) What is the measure, in degrees, of $\angle CDA$?

(a) 40   (b) 50   (c) 60   (d) 70   (e) 80
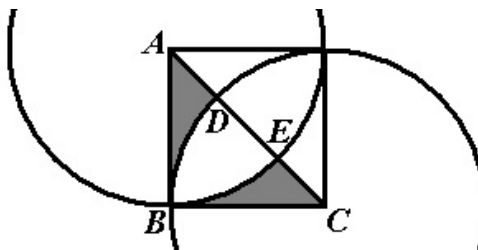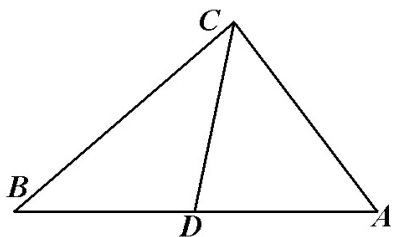
21. In the figure above right, $\triangle ABC$ is an isosceles right triangle, with $AB = BC = 2$. A circular arc of radius 2 with center C meets the hypotenuse at D, and a circular arc of radius 2 with center A meets the hypotenuse at E. To the nearest *HUNDREDTH* of a unit, what is the combined area of the two shaded regions?

(a) 0.59   (b) 0.65   (c) 0.71   (d) 0.78   (e) 0.86

22. In a hardware store, the cost of 14 screwdrivers, 6 hammers and 10 axes is 689.72 dollars. The cost of 15 screwdrivers, 5 hammers and 10 axes is 681.20 dollars. What is the cost, in dollars, of 1 screwdriver, 1 hammer, and 1 axe?

(a) 68.98   (b) 72.38   (c) 74.95   (d) 77.62   (e) 82.12

23. A right triangle, $\triangle ABC$, with hypotenuse AB, has side $AC = 15$. Altitude CH divides AB into segments AH and HB, with $HB = 16$. What is the area of $\triangle ABC$?

(a) 120   (b) 144   (c) 150   (d) 216   (e) $144\sqrt{5}$

24. Segment AB is both a diameter of a circle of radius 1 and a side of an equilateral triangle, $\triangle ABC$. The circle also intersects AC and BC at points D and E, respectively. What is the length of AE?

(a) $\dfrac{\sqrt{3}}{2}$ (b) $\dfrac{3}{2}$ (c) $\dfrac{5}{3}$ (d) $\sqrt{3}$ (e) $\dfrac{2+\sqrt{3}}{2}$

25. Using the digits 1, 2, 3, 4, 5, 6, 7, and 9, form four *two-digit* prime numbers, using each digit exactly once. What is the sum of the four prime numbers?

(a) 170   (b) 180   (c) 190   (d) 200   (e) 210

26. There are 12 marbles in a bag, all the same size. Five of the marbles are red, and the rest are blue. Jazzmine randomly removes two marbles from the bag, both at the same time. To the nearest *THOUSANDTH*, what is the probability that she chooses one red and one blue marble from the bag?

(a) 0.265   (b) 0.314   (c) 0.417   (d) 0.455   (e) 0.530

27. Robin's tractor can plow a one-acre field in two hours. Kerry's tractor can plow the same field in 36 minutes. If they worked together, how long, to the nearest *MINUTE*, would it take them to plow a seven-acre field?

(a) two hours and 42 minutes   (b) two hours and 48 minutes   (c) two hours and 58 minutes   (d) three hours and 6 minutes   (e) three hours and 14 minutes

28. We are happy you are at the SCSU Math Contest today—April 11, 2024! In honor of the occasion, find the number of positive integers that leave a remainder of 11 when divided into 2024.

(a) 5   (b) 6   (c) 8   (d) 11   (e) 15

29. The graph of $y = f(x)$ is a parabola that has a maximum at $(-1, 6)$ and contains the point $(2, 4)$. To the nearest *THOUSANDTH*, which of the following points is an x-intercept of the parabola?

(a) -6.515 (b) -6.196 (c) -5.893 (b) -5.605 (e) -5.330

30. Which of the following best describes the set of real-number solutions to the inequality

$$\left| x^2 + 6x + 1 \right| > 8?$$

(a) "all x's that are less than -3"

(b) "all x's that are greater than 1"

(c) "all x's that are greater than -7 AND less than 1"

(d) "all x's that are EITHER less than -3 OR greater than 1"

(e) "all x's that are EITHER less than -7 OR greater than 1"

31. My aunt's favorite positive number has three digits: 1, 6, and one other digit that I don't remember right now. I do remember that

- her favorite number is divisible by 9,
- it is *NOT* divisible by 4, and
- it has exactly three different *prime* factors.

What is the largest prime factor of my aunt's favorite positive number?

(a) 7   (b) 13   (c) 17   (d) 23   (e) 29

32. Abdi, Bernhard, Cua, and Dagmar are standing at the four corners of a rectangle; Abdi and Cua are diagonally opposite each other, and Bernhard and Dagmar are diagonally opposite each other. Ehawee is inside the rectangle. The distance from Ehawee to Abdi is ten feet, the distance from Ehawee to Bernhard is five feet, and the distance from Ehawee to Cua is eleven feet. The diagram below left illustrates the friends' positions, but it is not to scale. What is the distance from Ehawee to Dagmar?

(a) 9 feet   (b) $6\sqrt{3}$ feet   (c) 14 feet   (d) $9\sqrt{3}$ feet   (e) $8\sqrt{5}$ feet

33. In the diagram above right, Point P is interior to a square of sidelength a. (The diagram is not to scale.) Point P is equally distant from two consecutive vertices and from the side opposite those two vertices. Denote this common distance as "d." Then d is equal to which of the following?

(a) $\dfrac{3a}{8}$ (b) $\dfrac{a}{2}$ (c) $\dfrac{3a}{5}$ (d) $\dfrac{5a}{8}$ (e) $\dfrac{a\sqrt{2}}{2}$