2024 SCSU MATH CONTEST
11^{th} and 12^{th} GRADE

DIRECTIONS: Select the <u>BEST</u> completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. Determine the units digit of the decimal representation of 3^{2024}.

   (a) 1 (b) 3 (c) 7 (d) 9 (e) None of these

2. Find the sum of all natural numbers that are divisible by 3 and less than 2024.

   (a) $681,412$ (b) $681,751$ (c) $682,425$ (d) $1,362,825$ (e) $1,364,850$

3. Printer A, Printer B, and Printer C print at different speeds. If they work together, they can print an order of 5000 copies in 5 hours. Printer A would take 12 hours to complete the same order if working alone, and Printer B would take 15 hours if working alone. If all three printers are going to be used to complete the first half of the order, but then only Printer B and Printer C are used to finish the order, how many hours will it take to complete the entire order?

   (a) $\frac{40}{7}$ (b) $\frac{105}{17}$ (c) $\frac{145}{22}$ (d) $\frac{95}{14}$ (e) $\frac{515}{74}$

4. In 2022, a company sold 15% fewer shirts than they sold in 2021. In 2023, they sold 15% more than in 2022. In 2023, they sold 8,993 shirts. In 2024, they hope to sell 30% more shirts than they did in 2021. How many shirts do they need to sell in 2024 to reach their goal?

   (a) $10,166$ (b) $11,691$ (c) $11,700$ (d) $11,960$ (e) None of these

5. Let x and y be real numbers such that $y - x = 5$ and $-5 < x < 0$. Find the maximum value of the expression $(1 + x^{-1})(1 - y^{-1})$.

   (a) $\frac{1}{3}$ (b) $\frac{5}{21}$ (c) $\frac{16}{25}$ (d) $\frac{9}{25}$ (e) $\frac{1}{4}$

6. How many pairs (x, y) of natural numbers satisfy the equation $40x + 48y = 2024$?

   (a) 4 (b) 5 (c) 6 (d) 7 (e) 8

7. A tank contains 22 liters of antifreeze and water. This mixture is 40% antifreeze. How many liters of this mixture should be drained and replaced with pure antifreeze so that the mixture will be 50% antifreeze?

   (a) $2\frac{1}{3}$ (b) $2\frac{2}{3}$ (c) $3\frac{1}{5}$ (d) $3\frac{2}{3}$ (e) $4\frac{1}{2}$

8. Which of the following represents all of the solutions to the inequality $|x^2 + 6x + 1| > 8$?

   (a) $(-\infty, -3)$ (b) $(-\infty, -3) \cup (1, \infty)$ (c) $(-\infty, -7) \cup (1, \infty)$ (d) $(1, \infty)$ (e) $(-7, 1)$

9. In triangle ABC, angle A measures 120 degrees, side AC is 10 centimeters long, and side BC is three times the length of side AB. Find the length in centimeters of AB rounded to two decimal places.

   (a) 4.22 (b) 4.77 (c) 12.65 (d) 12.81 (e) None of these

10. A right circular cone has a base with a radius of 8 cm and a slant height with a length of 17 cm. Find the volume in cubic centimeters of the cone rounded to two decimal places.

    (a) 944.89 (b) 957.02 (c) 982.76 (d) 1005.31 (e) None of these

11. In how many ways can the letters in the word "HELLO" be arranged so that the vowels are not adjacent?

    (a) 24 (b) 36 (c) 60 (d) 72 (e) None of these

12. If it is known that $(3x - 4)$ is a factor of $6x^2 - 5x + k$, then what is the value of k?

    (a) -4 (b) -3 (c) 3 (d) 4 (e) None of these

13. A committee of 4 students is to be chosen from a group of 7 students: Abdul, Bethany, Carlos, Dominique, Eduardo, Faisal, and George. In how many ways can the committee be chosen if exactly one of Bethany or Faisal must serve?

(a) 10 (b) 20 (c) 36 (d) 40 (e) None of these

14. If $f(2x) = \dfrac{2}{2+x}$ for all $x > 0$, then find $2f(x)$.

(a) $\dfrac{2}{1+x}$ (b) $\dfrac{2}{2+x}$ (c) $\dfrac{4}{1+x}$ (d) $\dfrac{4}{2+x}$ (e) $\dfrac{8}{4+x}$

15. If $i^2 = -1$, then which of the following is a simplified form of $(i - i^{-1})^{-1}$?

(a) 0 (b) $-2i$ (c) $2i$ (d) $-\dfrac{i}{2}$ (e) $\dfrac{i}{2}$

16. If a, b, and c are real numbers satisfying the equations $2a + 13b + 3c = 90$ and $3a + 9b + c = 72$, then find the value of $\dfrac{3b+c}{a+2b}$.

(a) -1 (b) 0 (c) 1 (d) 2 (e) None of these

17. If a, b, and c are real numbers satisfying the equations $a+b+c = 1$ and $\dfrac{1}{a+1} + \dfrac{1}{b+3} + \dfrac{1}{c+5} = 0$, then find the value of $(a+1)^2 + (b+3)^2 + (c+5)^2$.

(a) 56 (b) 81 (c) 100 (d) 120 (e) 125

18. If a and b are integers such that $a^2 + b^2 = 29$, then what is the maximum value of $|a - b|$?

(a) -7 (b) 6 (c) 7 (d) 8 (e) None of these

19. If (x, y) is a solution to the system of equations $xy = 6$ and $x^2y + xy^2 + x + y = 63$, then what is the value of $x^2 + y^2$?

(a) 9 (b) 13 (c) $\frac{69-\sqrt{57}}{2}$ (d) 69 (e) 81

20. If $\log_4(\log_3(\log_2 x)) = 0$, then what is the value of $x^{-1/2}$?

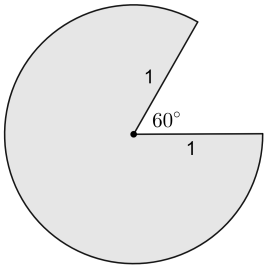
(a) $\frac{1}{2\sqrt{3}}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) None of these

21. If $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}$, then which of the following is true?

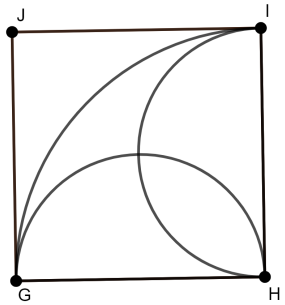
(a) $1 < x \leq 1.3$ (b) $1.3 < x \leq 1.5$ (c) $1.5 < x \leq 1.7$ (d) $1.7 < x \leq 2$ (e) $x > 2$

22. The radius of the circular sector shown below left is 1 cm, and the "mouth" has central angle 60°. Find the perimeter, in cm, of the circular sector.

(a) $\frac{5}{6}\pi + 2$ (b) $\pi + 2$ (c) $\frac{5}{3}\pi$ (d) 2π (e) $\frac{5}{3}\pi + 2$



Question 22



Question 23

23. The square shown above right has area 4 square inches. Find the area, in square inches, of the region inside the quarter circle but outside both of the half circles.

(a) $\frac{\pi}{4} - \frac{1}{2}$ (b) $\frac{\pi}{2} - 1$ (c) $\frac{3\pi}{4} - 1$ (d) $\frac{\pi}{2} + 1$ (e) $\frac{\pi}{4} + 2$

24. Two cards have the same probability of being chosen but one card is red on both sides while the other card is red on one side and blue on the other side. One of the cards is randomly chosen and placed on a table. If the visible side of the card on the table is red, then what is the probability that the hidden side of the card is also red?

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$ (e) $\frac{3}{4}$

25. Let $d = a^2+b^2+c^2$, where a and b are consecutive integers such that $c = ab$. Which of the following statements about $\sqrt{d}$ must be true?

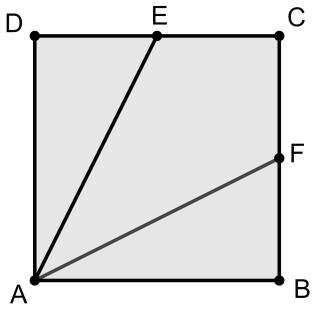
(a) $\sqrt{d}$ is an even integer for all values of a, b, and c that satisfy the conditions above.
(b) $\sqrt{d}$ is an even integer for some, but not all, values of a, b, and c that satisfy the conditions above.
(c) $\sqrt{d}$ is an odd integer for all values of a, b, and c that satisfy the conditions above.
(d) $\sqrt{d}$ is a rational number for some, but not all, values of a, b, and c that satisfy the conditions above.
(e) $\sqrt{d}$ is an irrational number for all values of a, b, and c that satisfy the conditions above.

26. Find the number of digits in the decimal expansion of $2^{20}5^{24}$.

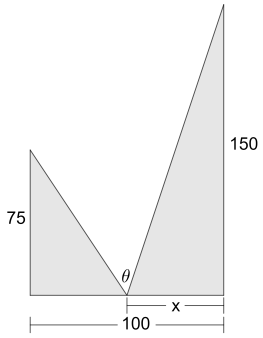
(a) 19 (b) 20 (c) 21 (d) 22 (e) 23

27. In the figure shown below left, suppose that $ABCD$ is a square and points E and F are the midpoints of sides DC and CB, respectively. What is the sine of $\angle EAF$?

(a) $\frac{2}{5}$ (b) $\frac{\sqrt{5}}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$ (e) None of these



Question 27



Question 28

28. If angle θ and quantity x are defined as shown in the figure above right, then which of the following is an expression for θ in terms of x?

(a) $\pi - \arctan\left(\dfrac{75}{100-x}\right) - \arctan\left(\dfrac{150}{x}\right)$

(b) $\pi + \arctan\left(\dfrac{75}{100-x}\right)$

(c) $\pi - \arctan\left(\dfrac{75}{100-x}\right) + \arctan\left(\dfrac{150}{x}\right)$

(d) $\arctan\left(\dfrac{75}{100-x}\right)$

(e) $\arctan\left(\dfrac{75}{100-x}\right) + \arctan\left(\dfrac{150}{x}\right)$

29. The sequence $\{a_n\}$ is defined by $a_1 = 2$ and $a_{n+1} = a_n + 2n$ for $n \geq 1$. Find a_{100}.

(a) 9900 (b) 9902 (c) 9904 (d) 10100 (e) 10102

30. A ball was floating in a lake when the lake froze. Without breaking the ice, when the ball is removed, it leaves a hole 24 cm across at the top and 8 cm deep. What is the radius of the ball in centimeters?

(a) 8 (b) 12 (c) 13 (d) $8\sqrt{3}$ (e) $6\sqrt{6}$

31. John and Lisa decide to play a game with a fair six-sided die whose faces are numbered 1 to 6. John rolls the die once and wins if he rolls a six. If he does not roll a six, then Lisa rolls the die once and wins if she rolls a six. The players continue to alternate rolls of the die until one of them rolls a six, at which point that player is named the winner and the game ends. What is the probability that John wins?

(a) $\frac{1}{2}$ (b) $\frac{6}{11}$ (c) $\frac{9}{16}$ (d) $\frac{3}{4}$ (e) 1

32. A surveyor determines that the angle of elevation to the top of a building from a point on the ground is 30.4°. He then moves back 55.4 ft and determines that the angle of elevation is now 23.2°. Find the height of the building to the nearest tenth of a foot.

(a) 31.7 (b) 32.5 (c) 88.1 (d) 91.5 (e) 150.2

33. If $\tan(\theta) = x$ and the terminal side of θ lies in Quadrant III, then which of the following is an expression for $\csc(\theta)$ in terms of x?

(a) $\sqrt{x^2+1}$ (b) $-\frac{1}{\sqrt{x^2+1}}$ (c) $\frac{x}{\sqrt{x^2+1}}$ (d) $\frac{\sqrt{x^2+1}}{x}$ (e) $-\frac{\sqrt{x^2+1}}{x}$

34. Consider the following snippet of pseudocode.

```
n = 9
i = 1
s = 3
t = 2
while i < n
    s = s + 2*i
    t = t + s - i
    i = i + 2
end while loop
print t
```

What value is printed after the code is executed?

(a) 30 (b) 35 (c) 53 (d) 58 (e) 102

35. Every inhabitant of the island of Smullyania is one of two types: either a Truthteller (who *always* tells the truth) or a Liar (who *always* lies). You meet two inhabitants of the island: Yolanda and Zander.

Zander says, "Yolanda is a Truthteller."

Yolanda says, "Well, exactly one of us is a Liar!"

Which of the following is correct?

(a) Both Yolanda and Zander are Truthtellers.

(b) Both Yolanda and Zander are Liars.

(c) Yolanda is a Truthteller, and Zander is a Liar.

(d) Yolanda is a Liar, and Zander is a Truthteller.

(e) Yolanda and Zander are of different types, but it is impossible to tell which is which.