DIRECTIONS: Select the **BEST** completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. The mean of a list of 15 numbers is equal to 42. The mean of 10 of these numbers is 26.9. What is the average of the remaining 5 numbers?

   A. 57.1   B. 63.4   C. 68.8   D. 72.2   E. 77.7

2. Mary is 20% older than Sally, and Sally is 40% younger than Danielle. The sum of Mary, Sally, and Danielle's ages is 23.2 years. How old will Mary be on her next birthday?

   A. 7   B. 8   C. 9   D. 10   E. 13

3. The set S is $\{1,2,3,4,5\}$. How many different subsets of S sum to exactly 9?

   A. 2   B. 3   C. 4   D. 5   E. 6

4. Suppose that $b \div c = 7$ and $a \div b = 3$. Find $\dfrac{a-b}{b-c}$.

   A. 7/3   B. 2.5   C. 2.6   D. 10/3   E. 3.5

5. Suppose that $\sqrt[5]{8} \cdot \sqrt[3]{16} = 2^n$. Find n.

   A. 15/8   B. 29/15   C. 7/3   D. 5/2   E. 27/10

6. The integer 1800 has how many positive integer divisors?

   A. 18   B. 24   C. 36   D. 45   E. 54

7. In right triangle ABC (with right angle $\angle ACB$), $AC = 15$, $\overline{CD}$ is an altitude of the triangle, and $DB = 16$.

   What is the area of $\triangle ABC$?

   A. 120   B. 144   C. 150   D. 216   E. $144\sqrt{5}$

8. For how many integers x does a triangle with side lengths 12, 25 and x have all of its angles acute?

   A. 5   B. 6   C. 7   D. 8   E. 9

9. Suppose that Lola has a list of positive integers, with repetitions allowed, and the average of the numbers on the list is 66. When one number, 80, is removed from the list, the average drops to 65. What is the largest possible number on the list?

   A. 140   B. 460   C. 844   D. 897   E. 910

10. Consider throwing three fair coins. What is the probability of getting exactly two heads?

    A. $\dfrac{1}{6}$ B. $\dfrac{3}{8}$ C. $\dfrac{4}{9}$ D. $\dfrac{2}{3}$ E. $\dfrac{3}{4}$

11. For what values of x is the expression $\log_8(\log_8(\log_8 x))$ defined?

    A. $(0,\infty)$ B. $(1,\infty)$ C. $(2,\infty)$ D. $(8,\infty)$ E. $(64,\infty)$

12. How many five-digit whole numbers that use each of the digits 0, 2, 3, 4, and 7 once are divisible by 4?

    A. 12　　　　　　　　B. 18　　　　　　　　C. 24　　　　　　　　D. 30　　　　　　　　E. 36

13. What is the sum of this infinite series?

$$\frac{3}{8} - \frac{5}{4} + \frac{3}{16} - \frac{5}{12} + \frac{3}{32} - \frac{5}{36} + \cdots$$

    A. -1.15　　　　　B. -1.125　　　　　C. -0.1875　　　　　D. 2.25　　　　　E. 2.65

14. The first four terms in my favorite arithmetic sequence are $x+y$, $x-y$, $x \cdot y$, and x/y, in that order. What is the fifth term of the sequence?

    A. $-9/8$　　　　　B. $-3/5$　　　　　C. 0　　　　　D. 27/20　　　　　E. 123/40

15. Which of the following is an equation for the set of all points in the xy-plane that are equidistant from the points $(-1, 4)$ and $(5, -2)$?

    A. $2x - y = 3$　　　B. $x - y = 1$　　　C. $x + y = 3$　　　D. $y = x^2 - 4x + 1$　　　E. $(x-2)^2 + (y-1)^2 = 18$

16. Two identical cylindrical tanks were full of water. Two different models of pumps were used to empty the tanks. The pumps were started at the same time, and each pumped at a constant rate. The first tank was emptied in 3 hours, and the second tank was emptied in 4 hours.

    At one moment during this process, the water depth in one tank was exactly twice the depth in the other tank. At that moment, how many minutes had the pumps been running?

    A. 60　　　　　　　　B. 72　　　　　　　　C. 120　　　　　　　　D. 132　　　　　　　　E. 144

17. Twenty-five people are arranged in a circle. Three are selected at random. What is the probability that no two of those selected were next to each other?

    A. 1/25　　　　　　B. 3/50　　　　　　C. 21/46　　　　　　D. 35/46　　　　　　E. 24/25

18. In rectangle $ABCD$ below left, $AD = 6$, $CD = 13$, and $\angle AEB$ is a right angle.

    What is the cosecant of $\angle DAE$?

    A. 0.554　　　　　　B. 0.832　　　　　　C. 1.202　　　　　　D. 1.248　　　　　　E. 1.803
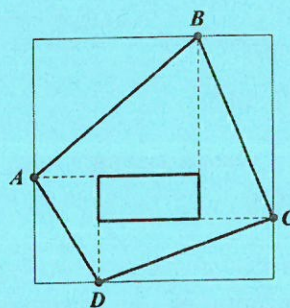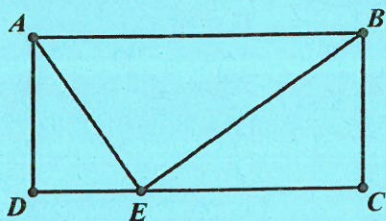
19. A rectangle of area 22 cm^2 is placed inside a square whose side lengths are 12 cm, with the sides of the rectangle parallel to sides of the squares. Perpendiculars are dropped from the sides of the square to the corners of the rectangle as shown in the diagram above right (which is not drawn to scale).

    What is the area of quadrilateral $ABCD$ in square centimeters?

    A. 72　　　　　　　　B. 83　　　　　　　　C. 86.4　　　　　　　D. 108　　　　　　　E. 132

20. Find the product of the roots of the equation $x^3 + 3x^2 = 4$.

    A. -4　　　　　　　B. 0　　　　　　　　C. 2　　　　　　　　D. 3　　　　　　　　E. 4

21. The solution of the equation $7^{x+7} = 8^x$ can be expressed in the form $x = \log_b \left(7^7\right)$. What is b?

    A. 7/15   B. 7/8   C. 8/7   D. 15/8   E. 15/7

22. Simplify

$$\sin\left(2\arctan x\right),$$

that is, $\sin(2\tan^{-1} x)$.

    A. $\dfrac{1}{\sqrt{1 - x^2}}$ B. $\dfrac{2x}{x^2 + 1}$ C. $\dfrac{x + \sqrt{x^2 - 1}}{1 + x\sqrt{x^2 - 1}}$ D. $\dfrac{\sqrt{x^2 + 1}}{x}$ E. $2x\sqrt{x^2 + 1}$

23. Let S be the set of all positive four-digit integers that have four distinct digits. How many of the integers in S are less than 1235?

    A. 14   B. 24   C. 30   D. 65   E. 111

24. Suppose x, y, and z are positive integers such that $x < y < z$ and

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1.$$

What is the value of $x + y + z$?

    A. 1   B. 4   C. 9   D. 11   E. No such integers exist.

25. How many integers k satisfy the inequality $6k^3 - 6k > k^4 - k^2$?

    A. 3   B. 4   C. 5   D. 6   E. infinitely many

26. The circle in the diagram below has radius 5, and ray $\overrightarrow{CA}$ is tangent to the circle at point A. The distance from point B to point C is 8. What is the distance from point A to point C?

    A. 10   B. 12   C. 13   D. 15   E. 16
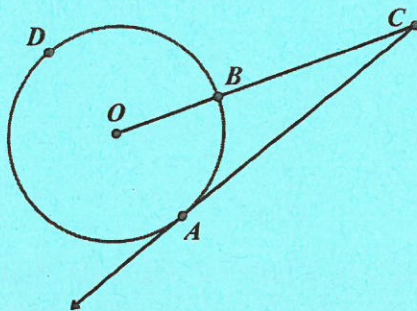
27. In the figure above, angle OCA measures $x°$. What is the measure, in degrees, of major arc ADB?

    A. $270 - x$ B. $270 + x$ C. $270 + 2x$ D. $360 - 2x$ E. $360 - x$

28. Let $x = \log_{10}(81)$ and $y = \log_{10}(25)$.

Suppose that a, b, and c are rational numbers such that $\log_{10}(6) = ax + by + c$.

What is the value of $a + b + c$?

    A. $-7/4$ B. $-5/6$ C. $3/4$ D. $5/6$ E. $7/6$

29. If the roots of the equation $x^2 + Bx + 1 = 0$ are the squares of the roots of the equation $x^2 + bx + 1 = 0$, which of the following expresses B in terms of b?

    A. $2 - b^2$ B. $1 - b^2$ C. b^2 D. $b^2 - 1$ E. $b^2 - 2$

30. Sally travels to work but must travel only north or east. Furthermore, because of construction, a section of road is unusable. Determine the number of ways Sally can get to work.

Sally's home → work (grid diagram)
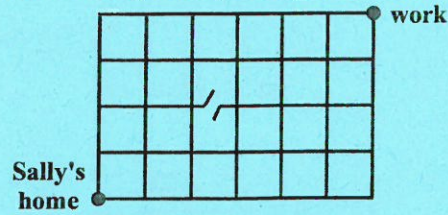
A. 60   B. 100   C. 150   D. 180   E. 210

31. My favorite polynomial has the form $x^5 + ax^4 + bx + c$, where the coefficients a, b, and c are real numbers. Which of the following is NOT possible for the roots of the polynomial?

  (a) three positive real roots, two negative real roots, no imaginary roots

  (b) three positive real roots, no negative real roots, two imaginary roots

  (c) two positive real roots, one negative real root, two imaginary roots

  (d) one positive real root, no negative real roots, four imaginary roots

  (e) no positive real roots, three negative real roots, two imaginary roots

32. Consider the following snippet of pseudocode:

```
s := 0
j := 1
m := 32
for i = 1 to 10
        if m > 1 then
                s := s + j
                j := j + 2
                m := m/2
        else
        end if
        next i
end for loop
print s
end
```

What value is printed for s?

A. 5   B. 9   C. 15   D. 21   E. 25

33. Every inhabitant of the island of Smulyania is either a *day-night* (telling only the truth during the day and only falsehoods during the night) or a *night-knight* (telling only falsehoods during the day and only the truth during the night). At noon one day, you meet three inhabitants of Smullyania who immediately make the following statements:

Aloysius says, "Exactly *two* of the three of us are night-knights."
Brunhilde says, "Exactly *one* of the three of us is a night-knight."
Chris chuckles and says, "Brunhilde said exactly the same thing twelve hours ago."

Which of the following can we conclude?

  (a) Aloysius is a day-knight; Brunhilde and Chris are both night-knights.

  (b) Aloysius is a night-night; Brunhilde is a day-knight; Chris is a night-knight.

  (c) Aloysius is a night-knight; Chris is a day-knight; it is impossible to know what type Brunhilde is.

  (d) Aloysius and Chris are both night-knights; it is impossible to know what type Brunhilde is

  (e) Brunhilde and Chris are both night-knights; it is impossible to know what type Aloysius is.