2009 SCSU MATH CONTEST
11th and 12th GRADE

DIRECTIONS: Select the <u>BEST</u> completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. When I walk to school, I average 90 steps per minute, with each of my steps 75 cm long. I take 16 minutes to get to school. My little brother Jack goes to school along the same route, averaging 100 steps a minute, with steps 60 cm long. How many minutes does Jack take to get to school?

   (a) $14\frac{2}{9}$ (b) 15   (c) 18   (d) 20   (e) $22\frac{2}{9}$

2. The rangefinder on Starbuck's *Viper* shows the enemy Cylon 4500 miles away. The *Viper* pursues the Cylon at 15000 MPH, and the Cylon retreat at 12000 MPH. How long will it take the *Viper* to catch up?

   (a) 1 hr, 30 min   (b) 1 hr, 40 min   (c) 2 hr, 20 min   (d) 2 hr, 45 min   (e) 3 hr, 20 min

3. What is the units digit of 3^{2009}?

   (a) 1   (b) 3   (c) 5   (d) 7   (e) 9

4. A right circular cylinder is 10 inches high and has a total surface area of 48π square inches. What is the *diameter* (in inches) of its base?

   (a) $\frac{2\sqrt{3}}{5}$ (b) $\frac{2\sqrt{6}}{5}$ (c) $2\sqrt{\frac{6}{5}}$ (d) 3   (e) 4

5. The mean (average) of the twelve integers in set A is 26, and the mean of the sixteen integers in set B is 40. What is the mean of all of the values together?

   (a) 28   (b) 30   (c) 32   (d) 34   (e) 36

6. An airplane that flies 113 m/sec in still air has a tailwind on one direction of a round trip. If the return trip takes 1.4 times as long as the trip out, what is the wind speed (in m/sec)?

   (a) $16\frac{1}{7}$ (b) $18\frac{5}{6}$ (c) $21\frac{3}{16}$ (d) $23\frac{13}{24}$ (e) $28\frac{1}{4}$

7. At Wired High School, 60% of the students own MP3 players, 50% own notebook computers, and 30% own both. Suppose that you pick a Wired student at random. What is the probability that the student owns at least one of these items?

   (a) 0.65   (b) 0.70   (c) 0.80   (d) 0.90   (e) 0.95

8. What is the coefficient on the $x^{12}y^7$ term in the full expansion of $(x + 2y)^{19}$?

   (a) 84   (b) 10752   (c) 50388   (d) 6449664   (e) 35831808

9. The centers of two circles of radius r are $3r$ units apart. A line is tangent to both circles and passes between them. What is the distance between the two points of tangency?

   (a) $\sqrt{2}r$ (b) $\frac{3}{2}r$ (c) $2r$ (d) $\sqrt{5}r$ (e) $3r$

10. A fair coin is flipped three times. What is the probability that "Heads" comes up on both the first and the third flip?

   (a) $\dfrac{1}{6}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{3}{8}$ (e) $\dfrac{1}{2}$

11. Let $\square ABCD$ be a square, and let M be the midpoint of BC and N the midpoint of CD. What is the sine of $\angle MAN$?

   (a) $\dfrac{1}{5}$ (b) $\dfrac{\sqrt{2}}{5}$ (c) $\dfrac{3}{5}$ (d) $\dfrac{4}{5}$ (e) $\dfrac{\sqrt{5}}{5}$

12. In Dr. Nemo's algebra class, 10 girls and 5 boys scored well enough on their exam to be eligible for a trip to the next Minnesota Wild hockey game. Dr. Nemo has three tickets to give away; to decide who gets them, she will randomly choose 2 of the girls and then randomly choose 1 of the boys. How many ways can she choose students for the trip?

   (a) 25   (b) 50   (c) 100   (d) 175   (e) 225

13. A set of ten positive integers has maximum value 8 and average 4. What is the *smallest* possible value of the *median* of this set of positive integers?

   (a) 2   (b) 2.5   (c) 3   (d) 3.5   (e) 4

14. Suppose that x and y are real numbers satisfying the equation $4y^2 + 4xy + x + 6 = 0$. What is the complete set of possible values for x?

   (a) $\{\, x \,:\, -2 \le x \le 3 \,\}$ (b) $\left\{\, x \,:\, \begin{matrix} x \le -2 \\ \text{OR } x \ge 3 \end{matrix} \,\right\}$ (c) $\{\, x \,:\, -3 \le x \le 2 \,\}$ (d) $\left\{\, x \,:\, \begin{matrix} x \le -3 \\ \text{OR } x \ge 2 \end{matrix} \,\right\}$ (e) $\left\{\, x \,:\, \begin{matrix} x \le 2 \\ \text{OR } x \ge 3 \end{matrix} \,\right\}$

15. Consider the following statement:

   Every child in this school who speaks French and Italian also speaks Russian.

   Which of the following is the negation (logical opposite) of the given statement?

   (a) "Every child speaks Russian, but not *both* French and Italian."
   (b) "No child speaks French, Italian, *and* Russian."
   (c) "No child who speaks French and Italian also speaks Russian."
   (d) "Some child speaks French and Italian, but does not speak Russian."
   (e) "Some child speaks Russian, but neither French *nor* Italian."

16. A philanthropist wants to give \$25 to each of several charities, but finds that she is exactly \$10 short. She notes that if she were to give only \$20 to each charity, she would have exactly \$25 left over. Finally she decides to drop one charity from her list. What is the greatest whole dollar amount that she can give to each of the remaining charities?

   (a) \$26   (b) \$27   (c) \$28   (d) \$29   (e) \$30

17. A fair six-sided die is rolled seven times. To four decimal places, what is the probability of rolling *at least one* 4?

   (a) 0.1667   (b) 0.3907   (c) 0.5714   (d) 0.7209   (e) 0.8452

18. A bag of coins contains pennies, dimes, and quarters (and *no other types of coin or currency*). There are twice as many dimes as pennies, and there are three times as many quarters as dimes. Which of the following *could* be the amount of money in the bag?

   (a) \$306.00   (b) \$333.00   (c) \$342.00   (d) \$348.00   (e) \$360.00

19. License plates in a certain U.S. state each have three letters followed by three digits. The first letter cannot be "Q" or "Z," and the last digit cannot be "0." Repetitions of letters and digits *are* allowed. How many different license plates are possible?

(a) 11197440   (b) 11232000   (c) 12119900   (d) 13055200   (e) 14601600

20. A rhombus is given with one diagonal twice the length of the other diagonal. Denote the *area* of the rhombus as A (square inches). In terms of A, how long (in inches) is a *side* of the rhombus?

(a) $\dfrac{\sqrt{A}}{4}$ (b) $\dfrac{\sqrt{2A}}{2}$ (c) $\dfrac{\sqrt{3A}}{2}$ (d) $\sqrt{A}$ (e) $\dfrac{\sqrt{5A}}{2}$

21. While visiting the Island of Liars and Truthtellers, you encounter two of its inhabitants, A and B.

A says, "B is a Liar."

B says, "We are not *both* Liars."

Liars *always* lie, and Truthtellers *always* tell the truth.

What can we conclude about A and/or B?

(a) A is a Liar, and   (b) A is a Liar, but   (c) B is a Liar, and   (d) B is a Liar, but   (e) Both are Liars.
B is a Truthteller.   nothing about B. A is a Truthteller.   nothing about A.

22. Suppose that a convex polygon has exactly three interior angles that are obtuse. What is the *maximum* possible number of sides of such a polygon?

(a) 4   (b) 5   (c) 6   (d) 8   (e) 10

23. An unfair coin comes up "Heads" $\dfrac{2}{3}$ of the time. You flip it 7 times. To four decimal places, what is the probability that it comes up "Heads" *exactly 4 times*?

(a) 0.0073   (b) 0.1296   (c) 0.2561   (d) 0.3810   (e) 0.5714

24. Equilateral triangles $\triangle ABC$ and $\triangle A'B'C'$ have parallel sides and the same center, and $\triangle A'B'C'$ lies on the interior of $\triangle ABC$, as in the diagram. (The diagram is *NOT* to scale.) The distance between side AB and side $A'B'$ equals $\dfrac{1}{6}$ of the altitude of $\triangle ABC$.
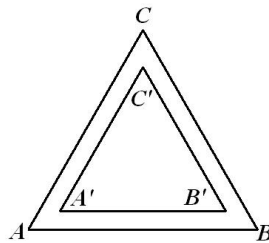
What is the ratio of the area of $\triangle A'B'C'$ to the area of $\triangle ABC$?

(a) $\dfrac{1}{36}$ (b) $\dfrac{1}{6}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{4}{9}$ (e) $\dfrac{\sqrt{3}}{4}$

(continued on next page...)

25. Suppose that x and y are real numbers such that $\dfrac{4^x}{2^{x+y}} = 8$ and $\dfrac{9^{x+y}}{3^{5y}} = 243$. Which of the following is equal to xy?

(a) -1 (b) $\dfrac{12}{5}$ (c) 4 (d) 6 (e) 12

26. Two tangents are drawn to a circle from an exterior point A; they touch the circle at points B and C. A third tangent to the circle intersects segment AB at P, intersects segment AC at R, and touches the circle at Q. Segment AB is 20 units long. What is the perimeter of $\triangle APR$?

(a) 42   (b) 41.012   (c) 40.5   (d) 40   (e) 39.875

27. The equation $x^{\log_{10} x} = 2009$ has two real roots. What is the product of these roots (to the nearest hundredth)?

(a) 1.00   (b) 2.14   (c) 2.73   (d) 4.66   (e) 7.96

28. A triangle, $\triangle ABC$, is inscribed in a semicircle of radius r so that the base AB is a diameter of the circle. Denote as s the *sum* of the lengths of the other two sides, AC and BC. Which of the following is true for all possible positions of C?

(a) $0 < s^2 < 4r^2$ (b) $s^2 = 4r^2$ (c) $4r^2 < s^2 \le 8r^2$ (d) $s^2 = 8r^2$ (e) $s^2 > 8r^2$

29. Let f be the function given by $f(x) = x^2$, and let h be any nonzero real number. The points $(2,4)$ and $(2+h, f(2+h))$ both lie on the graph of f. What is the slope of the line that goes through these two points?

(a) 1 (b) h (c) $2 - h$ (d) $4 + h$ (e) $\dfrac{4 + h^2}{2 + h}$

30. In $\triangle ABC$, we are given that $\angle C$ measures $105°$, $\angle B$ measures $45°$, and side AC is 2 units long. What is the length of side AB?

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{2} + 1$ (d) $\sqrt{3} + 1$ (e) $\sqrt{2} + \sqrt{3}$

31. What is the domain of the function f given by

$$f(x) = \sqrt{\log_2 \left(\frac{1}{\log_2 x} \right)} \ ?$$

(a) $\{\, x \ : \ x > 0 \,\}$ (b) $\{\, x \ : \ 0 < x < 1 \,\}$ (c) $\{\, x \ : \ 1 < x \le 2 \,\}$ (d) $\{\, x \ : \ x > 1 \,\}$ (e) $\{\, x \ : \ x \ge 2 \,\}$

32. The arch of a bridge is 18 feet high at its highest point and has a horizontal span of 60 feet, as shown in the diagram. (The diagram is *NOT* to scale.) The arch of the bridge is an arc of a circle, but it is *not* a semicircle.
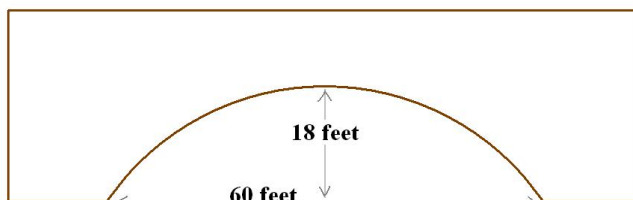
Find the radius of the circle that contains the arc.

(a) 32   (b) 34   (c) 36   (d) 38   (e) 40

(... OK, now you're done!)