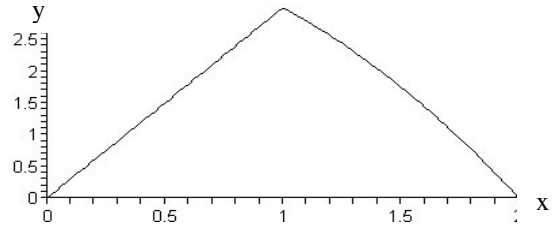


2005 SCSU MATH CONTEST
11th and 12th Grade Test

Scientific and graphic calculators are allowed. Symbolic calculators are not allowed.

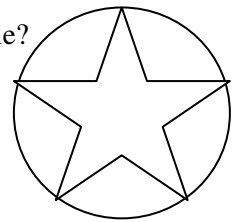
Directions: Select the one best response for each of the following problems.

1.) The graph below describes a salesperson's 1.5-hour journey from home in terms of time (x-axis) and distance from home (y-axis). Between time = 1 hour and time = 2 hours, the salesperson was:



- (a) Accelerating and getting farther from home
 (b) Decelerating and getting farther from home
 (c) Traveling at a constant speed and getting farther from home
 (d) Accelerating and getting closer to home
 (e) Decelerating and getting closer to home.

2.) What is the sum of the acute angles of the star shape which is inscribed in the circle?



- (a) 72° (b) 180° (c) 270° (d) 360° (e) 420°

3.) Let $f(x) = |x| - 1$. For how many x values is the function $f^{(4)}(x) = f(f(f(f(x))))$ zero?

- (a) 0 (b) 1 (c) 3 (d) 4 (e) 5

4.) When the polynomial $5x^3 + Mx + N$ is divided by $x^2 + x + 1$ the remainder is 0. What is $M + N$?

- (a) -3 (b) 5 (c) -5 (d) 15 (e) none of the above.

5.) If $f(x) = x + 4$ and $g(f(x)) = 2x + 1$, then $g(x) = ?$

- (a) $2x - 7$ (b) $8x + 3$ (c) $2x + 9$ (d) $2x^2 + 5x + 4$ (e) cannot be determined

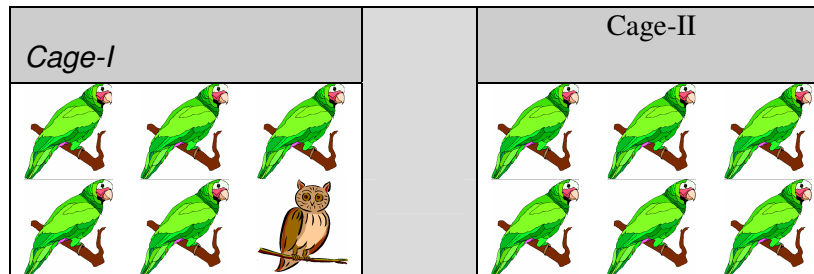
6.) A cube is made by soldering twelve 3-inch lengths of wire properly at the vertices of the cube. If a fly alights at one of the vertices and then walks along the edges, the greatest distance the fly could travel before coming to any vertex a second time, without retracing any wire, is less than:

- (a) 12 in. (b) 18 in. (c) 24 in. (d) 30 in. (e) 36 in.

7.) The simplest form of $1 + \frac{a}{1 - \frac{1}{1-a}}$ is:

- (a) a for $a \neq 1$ (b) a for $a \neq 0$ and $a \neq 1$ (c) $-a$ for $a \neq 1$ (d) $-a$ for $a \neq 0$ and $a \neq 1$ (e) 1 for $a \neq 1$

8.) Angie bought two cages of birds: Cage-I contains 5 parrots and 1 owl, and Cage-II contains 6 parrots.



Birds like to fly.

One day Angie forgot to lock both cages and two birds flew from Cage-I to Cage-II. Then two birds flew back from Cage-II to Cage-I. Assume that all birds have equal chance of flying. Find the probability the owl is still in Cage-I after all the flying.

- (a) 1/6 (b) 2/6 (c) 2/3 (d) 3/4 (e) none of the above

9.) The function f defined on the real numbers has the property that $f(f(x)) \cdot (1 + f(x)) = -f(x)$ for all x in the domain of f . If the number 3 is in the domain and range of f , what is $f(3)$?

- (a) -3 (b) -3/2 (c) -3/4 (d) 1/4 (e) 1/2

10.) The domain of the function $\log_{10}(\log_{10}(\log_{10}(x)))$ is

- (a) $\{x : x > 0\}$ (b) $\{x : x > 1\}$ (c) $\{x : x > 10\}$ (d) $\{x : x > 100\}$ (e) $\{x : 0 < x < 10\}$

11.) Assume x and y are both positive. For what y is $\ln(x + y) = \ln(x) + \ln(y)$?

- (a) $y = \frac{x}{1+x^2}$ (b) $y = \frac{x}{x-1}$ (c) $y = \frac{x^2}{1+x^2}$ (d) $y = \frac{1-x}{1+x}$ (e) $y = \frac{1+x^2}{1-x}$

12.) The floor of a square hall is tiled with unbroken square tiles. Along the two diagonals there are 125 tiles altogether. How many tiles are there in the hall?

- (a) 15625 (b) 15624 (c) 1537 (d) 2985 (e) 3969

13.) Six cards have on them the letters A, B, C, D, E, and F. If four cards are selected at random from these six and laid down in order, what is the probability that they will spell out “FADE”?

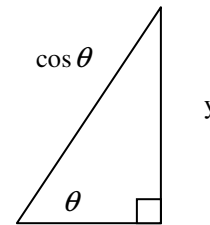
- (a) 1/720 (b) 1/360 (c) 1/180 (d) 1/45 (e) 1/6

14.) If x is an angle in the third quadrant, which of the following is equivalent to $\sec(x)$?

- (a) $\frac{1}{\sqrt{1-\sin^2 x}}$ (b) $\frac{1}{\sqrt{\cos x}}$ (c) $\frac{\sin(2x)}{\sqrt{\cos(x)}}$ (d) $\frac{\sqrt{2}}{\sqrt{1-\cos(2x)}}$ (e) $\frac{-1}{\sqrt{1-\sin^2(x)}}$

15.) In a right triangle with angle θ , the hypotenuse has length $\cos \theta$ as pictured. What is the length, y of the side opposite θ ?

- (a) $\frac{\sin(2\theta)}{2}$ (b) $\cot(\theta)$ (c) $\sin \theta$ (d) $\frac{\sqrt{1+\sin \theta}}{2}$ (e) $\frac{1}{2\sqrt{1-\sin \theta}}$



16.) How many different values of n are there such that n is a natural number and $n^2 - 20$ is a perfect square?

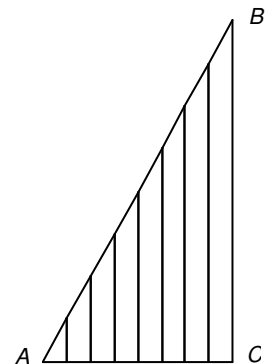
- (a) 1 (b) 2 (c) 3 (d) 4 (e) infinitely many

17.) If the square of a number consisting of two digits is decreased by the square of the number formed by reversing the digits, then the result is not always divisible by:

- (a) 9
 (b) 11
 (c) the product of the digits
 (d) the sum of the digits
 (e) the difference of the digits.

18.) Side AC of right triangle ABC is divided into 8 equal parts. Seven line segments parallel to BC are drawn to AB from the points of division (see diagram). If the length of BC is 10, then the sum of the lengths of the seven line segments is

- (a) 33 (b) 34 (c) 35 (d) 45 (e) none of the above



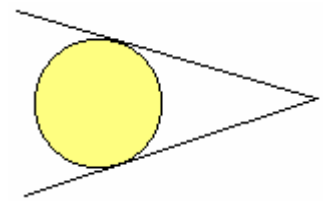
19.) The measure of an angle formed by the hands of a clock at 2:15 is:

- (a) 7.5° (b) 22.5° (c) 27.5° (d) 30° (e) 37.5°
-

20.) Suppose you have an investment which earns 10% the first year, 20% the second year, and 30% the third year. To the nearest 100th percent, what is its average rate of return (i.e., by what constant factor would your investment need to return each year in order to achieve the same total return)?

- (a) 18.44% (b) 19.72% (c) 20.00% (d) 20.66% (e) none of the above
-

21.) A ball with diameter 10 inches is pushed between two straight, hinged sticks, as shown. If the boundary of the ball is at minimum 18 inches away from the hinge, what is the angle between the two sticks at the hinge?



- (a) 12.6° (b) 13.5° (c) 15.0° (d) 25.1° (e) 30.0°
-

22.) What is the y-intercept of the line that is parallel to $y = 3x$, and which bisects the area of a rectangle with corners at $(0, 0)$, $(4, 0)$, $(4, 2)$, and $(0, 2)$?

- (a) $(0, -8)$ (b) $(0, -7)$ (c) $(0, -6)$ (d) $(0, -5)$ (e) $(0, -4)$
-

23.) What is the last digit of 2^{2005} ?

- (a) 0 (b) 2 (c) 4 (d) 5 (e) 6
-

24.) Fifteen Martians and ten Plutonians are at a party. Each Martian knows P of the Plutonians and each Plutonian knows M Martians. Assume M and P are both greater than 1 and that if a Martian knows a Plutonian, then that Plutonian knows that Martian, and vice versa. What is the smallest possible value for $M + P$?

- (a) 5 (b) 15 (c) 30 (d) 25 (e) 150
-

25.) Which number in base 9 is divisible by 8?

- (a) 7826_9 (b) 7716_9 (c) 4714_9 (d) 2345_9 (e) 1234_9
-

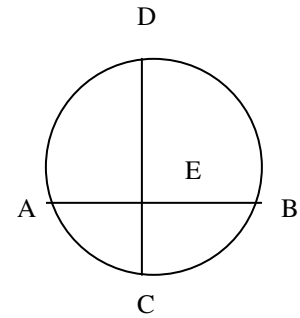
26.) The number $2 \log_2 \left(3^{\log_3 4} \right)$ simplifies as:

- (a) 12 (b) 24 (c) 72 (d) 4096 (e) 65536

27.) Define $a \otimes b = \begin{cases} a + b & \text{if } a < b \\ ab & \text{if } a = b \\ a - b & \text{if } a > b \end{cases}$. Find $8 \otimes ((4 \otimes 5) \otimes 1)$

- (a) 29 (b) 40 (c) 52 (d) 60 (e) 64

28.) Chords AB and CD in the circle intersect at E and are perpendicular to each other (see figure). If $AE = 2$, $EB = 6$, $ED = 3$, then the length of EC is:



- (a) 4 (b) 6 (c) 3 (d) $\sqrt{2}$ (e) $2\sqrt{2}$

29.) A regular octagon is to be formed by cutting equal isosceles right triangles from each of the corners of a square. If the square has sides of one unit, then the legs of each of the right triangles have length:

- (a) $\frac{2+\sqrt{2}}{3}$ (b) $\frac{2-\sqrt{2}}{3}$ (c) $\frac{2+\sqrt{2}}{2}$ (d) $\frac{1+\sqrt{2}}{3}$ (e) $\frac{2-\sqrt{2}}{2}$

30.) Two points are picked at random on a unit circle $x^2 + y^2 = 1$. What is the probability that the chord joining the two points has length at least 1?

- (a) 1/4 (b) 1/3 (c) 1/2 (d) 2/3 (e) 3/4

***Everyone who participates in the St. Cloud State Year 2005 Mathematics Contest is a winner!
Thank you and congratulations! Enjoy your day with us!***

