

2004 SCSU MATH CONTEST
9th and 10th GRADE

DIRECTIONS: Select the **BEST** completion or response from among those given. Scientific and graphing calculators are allowed. Symbolic calculators are not allowed.

1. A farmer, in order to save space, puts chickens and rabbits in the same cage. There are 35 heads and 94 feet (not counting the farmer). Let C be the number of chickens. What is the sum of the digits of C ?
(a) 3 (b) 5 (c) 7 (d) 8 (e) 11.
2. If the side of the square base of a pyramid is 30 meters and the height of the pyramid is 20 meters, then what is the volume of the pyramid in cubic meters?
(a) 4800 (b) 6000 (c) 7200 (d) 8400 (e) not enough information to calculate.
3. Simplify
$$\frac{x+2}{2-\frac{1}{1+\frac{1}{x}}}$$

(a) 1 (b) x (c) $x+1$ (d) $x-1$ (e) $\frac{1}{x+1}$.
4. The cost of 12 donuts and six éclairs is \$4.50 less than the cost of 12 éclairs and six donuts. By how much does the cost of one éclair exceed the cost of one donut?
(a) \$0.50 (b) \$0.60 (c) \$0.65 (d) \$0.75 (e) \$0.80.
5. Simplify $[-3^2 - (-2)^3][-2^3 - (-3)^2]$.
(a) -17 (b) -1 (c) 1 (d) 17 (e) 289.
6. The line that passes through the points $(-4, -2)$ and $(1, 8)$ also passes through the point $(x, 2004)$. Find x .
(a) 999 (b) 1000 (c) 1009.5 (d) 1996 (e) 3993.
7. $ABCD$ is a square. E , F , G , and H are the midpoints of AB , BC , CD , and AD , respectively. What is the ratio of the area of square $ABCD$ to the square $EFGH$?
(a) 4 : 1 (b) 3 : 1 (c) 2 : 1 (d) 3 : 2 (e) none of these.
8. Each year, a computer is worth 15% less than it was worth the previous year. What will a \$2004 computer bought in the year 2004 be worth in the year 2010?
(a) \$0.02 (b) \$20.40 (c) \$200.40 (d) \$755.81 (e) \$1703.40.
9. How many of the following numbers are rational: π , $3/5$, $\sqrt{2}$, 0.3333 , $0.\overline{571428}$, and $\sqrt{4}$.
(a) 2 (b) 3 (c) 4 (d) 5 (e) 6.
10. If the six-digit number $348B20$ is divisible by 45, then B could be which of the numbers below?
(a) 1 (b) 3 (c) 4 (d) 7 (e) 9.
11. Which of the expressions below is a factor of $16t^2 - 74t + 40$?
(a) $t - 5$ (b) $8t - 4$ (c) $8t - 5$ (d) $16t - 4$ (e) $16t - 5$.
12. The five fastest runners in a race are awarded money. The total of the award is \$600. The fastest runner gets \$30 more than the second, the second gets \$30 more than the third, and so on. How much does the fourth fastest runner get?
(a) \$45 (b) \$60 (c) \$75 (d) \$90 (e) \$100.

13. If two factors of $2x^3 - hx + k$ are $x + 2$ and $x - 1$, then what is $h + k$?
- (a) -6 (b) 7 (c) 8 (d) 9 (e) 10 .
14. A circle is inscribed in a square. About what percentage of the area of the square is *outside* of the circle?
- (a) 7.5% (b) 11.5% (c) 21.5% (d) 65.5% (e) 78.5% .
15. Of the 2000 animals at the Middletown Zoo, 2% of them are rabbits. How many rabbits must be added to the zoo to increase the percentage of rabbits to 20% ?
- (a) 40 (b) 45 (c) 360 (d) 400 (e) 450 .
16. A farmer encloses a field that borders a long building as shown in the figure below. He uses three times as much fencing material on the side opposite the building as he uses on the other sides combined. If the farmer used a total of 160 feet of fencing material, what is the area of the enclosed field in square yards?
- (a) $266\frac{2}{3}$ (b) 300 (c) $355\frac{5}{9}$ (d) $341\frac{1}{3}$ (e) $533\frac{1}{3}$.
17. In the expression x^2y , the values of x and y are each decreased by 25% . The entire expression decreases by how much?
- (a) decreased by 50% (b) decreased by 75% (c) decreased by $\frac{17}{64}$ (d) decreased by $\frac{27}{64}$ (e) decreased by $\frac{37}{64}$.
18. The graphs of $x + y = 10$ and $x^2 + y = 10$ intersect in two points. The distance between these two points is:
- (a) less than 1 (b) 1 (c) between 1 and 2 (d) 2 (e) more than 2.
19. What is the area of a triangle whose sides are of length 12, 16, and 20?
- (a) 28 (b) 96 (c) 120 (d) 192 (e) not enough information to calculate.
20. A T puzzle is made up of four pieces, as illustrated below but not shown to scale. C , H , and I are collinear and $GH = EF$. B is the midpoint of AC and E is the midpoint of FD . What is the area of the piece marked x ?
- (a) 1 (b) 1.5 (c) 2 (d) 2.5 (e) 3.
21. Solve the equation $3^x - 3^{x-1} = 162$ for x .
- (a) 3.5 (b) 4 (c) 5 (d) 5.5 (e) none of these.
22. Suppose $f(2x) = x^2 - 4x + 8$. What is $f(x)$?
- (a) $\frac{x^2}{4} - 2x + 8$ (b) $\frac{x^2}{4} - 2x + 4$ (c) $\frac{x^2}{2} - 2x + 8$ (d) $\frac{x^2}{2} - 2x + 4$ (e) $2x^2 - 8x + 16$.
23. The base of a triangle is 15 inches long. Two line segments parallel to the base divide the triangle into three equal areas. What is the length of the line segment that's *closer* to the base?
- (a) $4\sqrt{3}$ inches (b) $5\sqrt{6}$ inches (c) 7.5 inches (d) 8 inches (e) 10 inches.
24. What is the area of the triangle formed by lines $y = 3x + 2$, $y = -3x + 2$, and $y = -2$?
- (a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{8}{3}$ (d) $\frac{16}{3}$ (e) none of these.

25. If $f(x) = x^2 - 4x$, find $f(x - 3)$.
- (a) $x - 3^2 - 4x - 3$ (b) $x^2 - 6x + 9 - 4x$ (c) $x^2 - 10x + 21$ (d) $x^2 - 6x + 9$ (e) $x^2 - 4x - 3$.
26. What is the least value of y that satisfies the inequality $|4 + x| + |5 + y| \leq 100$?
- (a) -125 (b) -105 (c) -55 (d) 0 (e) 20 .
27. Sue leaves Appleton and drives 65 miles per hour to visit her sister Jane in Barryville 100 miles away. However, simultaneously, Jane leaves Barryville to visit Sue, driving 55 miles per hour. Approximately how far are they from Appleton when they pass one another?
- (a) 52 miles (b) 54 miles (c) 55 miles (d) 57 miles (e) 60 miles.
28. When a company produces 500 widgets, it charges \$3.30 each. When producing 750, it charges \$2.80. If price is a linear function of the number of widgets produced, then what price should be charged for each widget if 900 are produced?
- (a) \$2.50 (b) \$2.30 (c) \$2.60 (d) \$2.45 (e) \$2.55.
29. Which of the following depicts the commutative property of addition?
- (a) $(a + b) + c = a + (b + c)$ (b) $x(y + z) = xy + xz$ (c) $mn = nm$ (d) $rs + ps = (r + p)s$ (e) $a + b = b + a$.
30. Three fair six-sided dice are rolled. What is the probability that the product of the three numbers showing will be odd?
- (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{6}$ (d) $\frac{1}{8}$ (e) $\frac{1}{10}$.
31. If $3^{3y} = 125$, then 3^{-y} equals
- (a) $-\frac{1}{5}$ (b) $\frac{1}{9}$ (c) $\frac{1}{375}$ (d) $\frac{1}{125}$ (e) $\frac{1}{5}$.
32. Suppose that x is a positive number that satisfies the equation $x = \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}$ (here “...” means that the pattern continues forever). What is the value of x ?
- (a) 1 (b) $\sqrt{2}$ (c) $\sqrt{2 + \sqrt{2 + \sqrt{2}}}$ (d) $1 + \sqrt{2}$ (e) 2.
33. Four fair coins are tossed. What is the probability that at least three of them land heads?
- (a) $\frac{5}{16}$ (b) $\frac{3}{16}$ (c) $\frac{5}{32}$ (d) $\frac{4}{16}$ (e) $\frac{7}{32}$.
34. Let f be the function given by $f(x) = \frac{1}{\sqrt{1-x}}$. Suppose that A is a number such that $f(A) = 3$. Then what is $f(1 - A)$?
- (a) $\frac{\sqrt{3}}{3}$ (b) $\frac{3\sqrt{2}}{4}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{1}{9}$ (e) $\frac{8}{9}$.
35. I have a friend who always lies on Mondays, Wednesdays, and Thursdays, but always tells the truth on the other days of the week. One morning he said “Tomorrow is *NOT* one of my lying days.” Later in the day he said “The day after tomorrow *IS* one of my lying days.” What day of the week was it when he said this?
- (a) either Monday or Saturday (b) either Monday or Wednesday (c) either Tuesday or Friday (d) either Wednesday or Friday (e) either Wednesday or Saturday.
36. In the diagram below, point O is the center of the circle, and the points A , B , and C are on the circle. If the measure of $\angle AOC$ is 84 degrees, then what is the measure of $\angle ABC$?
- (a) 30 degrees (b) 33 degrees (c) 36 degrees (d) 42 degrees (e) 48 degrees.