

# **Leadership and Group Size: An Experiment**

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## Introduction

Free riding and coordination failure are two common problems in collective actions. Recent theoretical and experimental work has shown that leading by example can improve collective outcomes if a leader's effort induces followers to make an effort as well.<sup>1</sup> Potters, et al. (2007, PSV) consider a voluntary contribution mechanism in which two group members choose whether to make a costly contribution to a group activity which benefits both group members. The benefit is uncertain: it may be low, medium or high. Cooperation is efficient when the benefit is medium or high, but not low. The free riding problem can prevent cooperation when the benefit is medium. In PSV, a leader-follower setting in which only the leader knows the benefit can, theoretically, overcome the free-riding problem: the leader contributes whenever the benefit is medium or high and the follower also contributes. PSV show that, although efficient cooperation is not 100% as predicted, an impressive percentage of the leaders contribute whenever it is efficient to do so (almost 100% when the benefit is high and 75% when the benefit is medium) and 80.6% of the followers mimic their leader's lead.

Komai, et al. (2007, KGD) provide experimental results consistent with PSV in a more complicated setting. In PSV, subject's payoffs are linear in actions and the leader is followed by only one follower; PSV thus ignore the interaction among the followers and the possibility of the coordination problem. In KGD, subjects' payoffs are non-linear in actions and leaders are followed by two followers; thus KGD addresses both the interaction among the followers and the possible coordination problem.

We consider the same leading by example mechanism as in PSV and KGD but show how its effectiveness changes with group size. We show that leading by example loses its

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<sup>1</sup> Leaders' effort may convey relevant information to followers, followers may have non-pecuniary motives such as reciprocity, or both. See Potters, et al. (2007) and Komai, et al. (2007) for a literature review.

effectiveness as groups become larger; an issue which is not addressed in the literature to our knowledge.

## Experiment

We consider a simple single-shot collective action game that exhibits both problems of free riding and coordination failure. Consider a group of size  $m$ . Each player has an endowment of \$10 and decides whether to invest it in a joint investment project. An investor loses the \$10 but earns  $(x+10)q$ ;  $x$  is a random return (uniformly distributed over the interval  $X=[-15,15]$ ), and  $q$  is the fraction of players who invest in the project including the investor. A non-investor keeps the \$10 and earns a payoff of  $(\frac{2}{3}x+5)q^n$ , where  $q^n=q-\frac{1}{m}$ . A project is good if  $x>0$  and bad if  $x<0$ . In our setting, uninformed players (about  $x$ ) are unwilling to invest. No player is willing to invest alone. There is increasing returns to investment when  $x>0$ . There is an incentive to free-ride on the others (but if  $x$  is very high, a player prefers to invest if he believes everybody else invests). Finally, whenever  $x>0$  full cooperation is efficient.<sup>2</sup>

Suppose  $m=3$ ; then for  $6<x\leq 15$ , the coordination problem, and for  $0<x<6$  the free-riding problem can prevent efficient group cooperation.<sup>3</sup> Now let  $m=9$ ; then for  $10.9<x\leq 15$ , the coordination problem, and for  $0<x<10.9$  the free-riding problem undermine efficiency.<sup>4</sup>

Consider the following leader-follower setting. Only the leader has access to  $x$ ; other group members are aware only about how  $x$  is distributed. The leader decides first whether to invest. The followers, after observing the leader's action, make their investment decisions

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<sup>2</sup> This model can be appropriately applied to ad hoc committees established to work on a temporary, single-shot tasks such as implementing a restructuring plan, developing a new product or adopting a new procedure or technology.

<sup>3</sup> At  $x=6$  investment is weakly dominated.

<sup>4</sup> At  $x=10.9$  investment is weakly dominated.

simultaneously. This setting can theoretically alleviate both the coordination and the free-riding problems; there always exists a trivial equilibrium in which no one ever participates; Alternatively, there exists a productive equilibrium in which the leader participates when it is efficient to do so (whenever  $x > 0$ ) and followers mimic the leader iff  $E_x[(x+10)q - (\frac{2}{3}x+5)q^n - 10 | x > 0] > 0$  (i.e., iff a follower expects to gain from investment after observing the leader's investment signal).<sup>5</sup>

Our experiment is based on the above model and consists of two treatments: T3 ( $m=3$ ) and T9 ( $m=9$ ). In both treatments, a group is randomly assigned one of the following three payoff scenarios, each occurring with equal probability. Scenario 1 ( $x=12$ ) represents the coordination problem; Scenario 2 ( $x=3$ ) represents the free riding problem; and Scenario 3 ( $x=-10$ ) represents a bad project (see Table 1).<sup>6</sup> In both treatments, only the leader observes the scenario; followers only know the possible scenarios and their likelihood.

Four sessions of 15 (18) subjects for T3 (T9) were conducted. There was no show up fee. In T3 (T9) subjects were seated, in a room, separately in 3 rows of 5 (6). In T3 (T9) we created 5 (2) groups of 3 (9).

After subjects sign a consent form, experimenters read aloud the instructions. Subjects are tested to make sure they understand the game. Each session consists of 10 rounds; subjects begin each round with a \$10 endowment. In T3 (T9) a subject is randomly grouped with two (eight) other subjects. Groups are reformed each round and no subject is grouped with the same two (eight) people in more than one round. Leader/follower roles vary each round. This re-

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<sup>5</sup> In equilibrium increasing returns to participation implies that all followers choose the same strategy. Proof is available upon request.

<sup>6</sup> Numbers are rounded but the payoff structure still preserves all the theoretical characteristics.

matching of subjects was introduced by Andreoni (1988) to balance the desire to test a single-shot prediction with the need for repeated experience by the subjects.

Payoff scenarios are randomly determined at the beginning of each round and vary across rounds. A scenario is the same for all group members but different across groups. At the end of each round subjects' individual earnings are calculated and announced privately. Subjects are informed that their final earnings will be those of only one round chosen randomly (using a die) at the end and therefore they should make the best decision possible in each round. Finally, subjects complete survey forms and receive their earnings privately using an identification number which is assigned to them at the outset of the session.<sup>7</sup> Sessions take approximately 90 minutes. Subjects earn \$13.76 (\$11.01) in T3 (T9).

A total of 132 subjects participated. Table 2 reports subjects' investment rates by treatment (by round and in total), for Scenarios 1 and 2.<sup>8</sup> Consider Scenario 1. A total of 60 (70) subjects in T3 (T9) were in Scenario 1. Table 2 shows that the investment rate in T3 is significantly higher than the investment rate in T9 (86.9% and 50.6%, respectively). A Mann-Whitney rank-sum test shows that this difference is significant (p-value = 0.000).<sup>9</sup> A total of 60 (72) subjects in T3 (T9) were in Scenario 2. Subjects' total investment rates in T3 are again significantly higher than in T9 (75.4% and 24.0%, respectively, p-value = 0.000).

Consider leaders and followers separately. According to Table 3, 100% (79.4%) of the leaders invested in T3 (T9). This difference is, however, not significant (p-value = 0.159). In

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<sup>7</sup> The survey includes an item to determine the clarity of instructions. Subjects' answers are on a five-point Likert scale (1= not clear; 5= very clear). The average response was above 4.5 with no significant difference across treatments.

<sup>8</sup> For Scenario 3, investment is neither individually nor socially optimal. Our results show few investments in Scenario 3 and no significant difference across treatments.

<sup>9</sup> Our method is the same as Andreoni (1995). We calculate the investment rate of each subject and then rank subjects by their investment rates in the joint sample; under the null hypothesis of no difference between the two treatments, the sum of the ranks should be equal across treatments. All p-values reported are for Mann-Whitney rank sum tests.

Scenario 2, the difference is more pronounced (85.0% in T3 versus 42.9% in T9, p-value= 0.001). This suggests that leaders in T9 did not expect their followers to follow them as much as leaders in T3. This expectation significantly reduced the leaders' investment in Scenario 2 (but not as much in Scenario 1, because followers' refusal to follow does not hurt leaders in Scenario 1 as much as it does in Scenario 2). After observing their leaders' investment, followers' behavior indicates the leaders' instincts were correct: 82.9% of the followers invested in T3 versus 46.9% in T9 (p-value= 0.000).<sup>10</sup>

## **Conclusion**

The discrepancy between the leader's incentives and those of an individual follower increases with group size. Followers become more marginalized and thus more eager to free ride and less eager to invest [in T3 (T9) a follower is 1/3 (1/9) of the group], while the leader is still interested in investing both in T3 and T9 in Scenarios 1 and 2 (if followed). Thus, the leader of a large group may no longer represent the interest of an individual follower. As a result, the leader's persuasive signal may become too weak to overcome the free riding tendencies of an individual follower. Followers (also expecting the same behavior from other followers) do not follow the leader's lead in T9 as much as they do in T3. Leaders rationally understand this and do not initiate investments in T9 as often as they do in T3, in particular in Scenario 2, because followers' refusal to follow hurts leaders more in Scenario 2 than it does in Scenario 1.

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<sup>10</sup> We did not separate followers by scenario, since they cannot directly observe which scenario they are in.

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Table 1: Payoff Scenarios  
Scenario 1  
(Scenario 2)  
**Scenario 3**

	m=3		m=9	
	Invest	Not Invest	Invest	Not Invest
All	22 (13) <b>0</b>	–	22 (13) <b>0</b>	–
8	...	...	20 (12) <b>0</b>	21 (16) <b>8.5</b>
7	...	...	17 (9) <b>0</b>	20 (15) <b>8.7</b>
6	...	...	15 (7) <b>0</b>	18 (14) <b>8.9</b>
5	...	...	12 (6) <b>0</b>	17 (13.5) <b>9</b>
4	...	...	10 (5) <b>0</b>	15 (13) <b>9.2</b>
3	...	...	7 (4) <b>0</b>	14 (12) <b>9.4</b>
2	15 (9) <b>0</b>	19 (15) <b>8</b>	5 (3) <b>0</b>	12 (11.5) <b>9.6</b>
1	7 (5) <b>0</b>	15 (12) <b>9</b>	3 (2) <b>0</b>	11 (11) <b>9.8</b>
0	–	10 (10) <b>10</b>	–	10 (10) <b>10</b>

Table 2: Investment Rates

	Scenario 1										
Round	1	2	3	4	5	6	7	8	9	10	Total
T3	91.7%	79.2%	91.7%	83.3%	95.8%	95.8%	87.5%	83.3%	83.3%	100.0%	86.9%
T9	44.4%	...	61.1%	61.1%	55.6%	55.6%	...	38.9%	41.7%	...	50.6%
	Scenario 2										
T3	58.3%	75.0%	83.3%	72.9%	66.7%	100.0%	62.5%	83.3%	79.2%	70.8%	75.4%
T9	41.7%	66.7%	33.3%	27.8%	33.3%	5.6%	13.9%	16.7%	11.1%	15.3%	24.0%

Table 3: Leaders' Investment Rates

	Scenario 1										
Round	1	2	3	4	5	6	7	8	9	10	Total
T3	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
T9	50.0%	...	100.0%	100.0%	100.0%	100.0%	...	75.0%	75.0%	...	79.4%
	Scenario 2										
T3	62.5%	75.0%	87.5%	87.5%	75.0%	100.0%	62.5%	87.5%	87.5%	87.5%	85.0%
T9	50.0%	100.0%	50.0%	50.0%	50.0%	0.0%	25.0%	0.0%	25.0%	37.5%	42.9%

Table 4: Followers' Investment Rates (Given Leader Invested)

	All Scenarios										
Round	1	2	3	4	5	6	7	8	9	10	Total
T3	88.9%	77.3%	90.9%	72.2%	88.2%	95.8%	84.6%	83.3%	76.7%	77.3%	82.9%
T9	81.3%	62.5%	58.3%	52.5%	50.0%	50.0%	37.5%	41.7%	31.3%	29.2%	46.9%