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A Theory of Transition to a Better Technology*

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ABSTRACT

This paper builds a model of transition following economic reforms and analyzes the different experiences of two Central European economies after 1990. East Germany started its transition with rapid growth in output per working-age person and experienced a dramatic increase in its very low initial capital income share of output. Poland experienced low growth in output per working-age person while maintaining a fairly constant capital income share. Reform is modeled as gaining access to a higher productivity technology, embodied in new plants. As new, high productivity plants are built, the existing low productivity plants decrease their production and eventually shut down. During this process, the capital income share varies. Two policies are incorporated in the model: transfers from the rest of the world and wage increases due to political pressure. The model quantitatively captures both the East German and Polish experience.

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1 Introduction

This paper builds a theory of transition of an economy that gains access to a better, higher productivity technology. The theory is used to analyze the different transition experiences of two Central European economies after 1990.

The economic reforms of the late 1980s and early 1990s in Central Europe started a restructuring of the centrally planned economies that brought access to the technologies of advanced industrialized economies. The period of transition that followed showed similarities as well as differences between economies. Following a few years of recession most economies experienced positive growth in output per working-age person up to present times.[1] However, the growth experiences have been quite different across economies. For example, between 1991 and 1996, gross domestic product per working-age person showed an annual average growth rate of 1.7 percent in the Czech Republic and the Slovak Republic, 4.2 percent in Poland and 7.6 percent in East Germany.[2] In addition, there are interesting differences in the distribution of income between capital and labor over the transition. In East Germany, the share of total income attributed to capital showed a dramatic increase, from 12 percent of total income in 1991, to 26 percent in 1996. Thereafter, it averaged 29 percent. However, in the Czech Republic, the Slovak Republic, and Poland, the capital income share was roughly constant.

The facts above are puzzling from the perspective of a standard neoclassical growth model where economic reforms are modeled as an exogenous increase in total factor productivity. The predictions of this model are fast initial output growth and a constant capital income share, for all economies. This paper proposes a new theory to understand the different transition experiences of the Central European economies.

The paper focuses on the transition experiences of two economies: East Germany and Poland. It addresses the following questions: What accounts for the different growth in output per working-age person in East Germany and Poland? What accounts for the different evolution of the capital income share in East Germany and Poland?

[1] Exceptions include Ukraine and Romania. Ukraine experienced a prolonged recession, and showed positive growth in output per capita only in 1999, as documented by the Groningen Growth and Development Center. Romania showed positive growth during the mid 1990s, followed by recession years in the late 1990s.

[2] Unless otherwise stated, data on East Germany cover the five eastern area states of Germany, excluding East Berlin.

To address these questions, I consider a dynamic general equilibrium model that generates time-varying factor income shares. Reform is modeled as gaining access to a higher productivity technology embodied in new plants. This is motivated by the fact that following the reforms in the late 1980s, the Central European economies have removed barriers to technology from industrialized economies that were previously in place. In the model, as new, high productivity plants are built, the existing low productivity plants decrease their production and eventually shut down. During this process, the low productivity plants have a time-varying profit share. This is reflected in a capital income share for the economy that varies during the transition to a new steady state.

Two policies are incorporated in the model. The first policy is transfers received by the economies in transition from the rest of the world. The economic restructuring of the centrally planned economies has been eased somewhat by various types of aid. Poland benefited from transfers from the European Union. East Germany benefited from transfers from the European Union, and from West Germany. The second policy is wage increases due to political pressure or union power. Each of these policies were different across the two economies. First, East Germany received larger transfers from the rest of the world compared to Poland. Second, wage increases during the initial years of transition were larger in East Germany than in Poland. In the model, transfers allow for higher investment in the plants with the better technology, thus leading to faster growth. Moreover, increases in wages reduce the profit share of low productivity plants, thus leading to a low capital income share. Hence, during the transtion to the better technology, policies of different magnitude contribute to different transition experiences for the two economies.

The model is parameterized to match key facts of the East German economy. I find that the model accounts for 63 percent of the growth in output per working-age person in East Germany over the period 1991 to 1996. The model captures a rapid growth in output due to the presence of transfers. Furthermore, the model captures a very low capital income share, of 22 percent, for East Germany in 1991. Without policies, the model generates small changes in the capital income share, due to a time-varying profit share at the low productivity plants. With the introduction of the two policies, the profit share of the low productivity plants lowers further, and hence yields a very low capital income share.

Using the model to analyze the experience of Poland, I find that it accounts for 83 percent of the growth in output per working-age person over the period 1991 to 1996. Moreover, the model predicts a roughly constant capital income share for Poland. The capital income share in the model varies over time; however, these variations are very small given the small magnitude of policies. When comparing the experience of East Germany and Poland, I find that the model accounts for 45 percent of the differences in output growth over the period 1991 to 1996. The main driving force behind this result is the presence of large tranfers to East Germany that allow for a faster accumulation of high productivity capital and hence faster growth.

The theory used in this paper draws on Hansen and Prescott (2005). A feature of their real business cycle model is a counter-cyclical labor income share. In this paper, I use a similar plant level technology in a model with two types of plants and no aggregate technology shocks. This model is able to capture low frequency changes in factor income shares.

Various aspects of transitions have been addressed in the works of other researchers. A recent World Bank report (2002) incorporates a summary of cross-country empirical literature on growth in transition economies.[3] Researchers in this literature use empirical methods to identify factors that contribute to differences in the growth experiences of transition economies. A key finding is that different policies play a large role in explaining differences in growth experiences.[4] Among theoretical work, Atkeson and Kehoe (1993) build a model that predicts recessions as the initial phase of transition following economic reforms. The current work differs from previous literature in that it is a quantitative study of the growth experiences following the initial recession period.

The paper is organized as follows. Section 2 presents data on the transitions of East Germany and Poland. Section 3 describes the model economy. Section 4 describes the model's implications for output growth and the capital income share. Numerical experiments for East Germany and Poland are presented in Section 5. Section 6 concludes.

[3] See World Bank (2002), pages 16-20.

[4] See for example Fischer, Sahay and Végh (1996). Among other factors, they find foreign aid during transitions to be conducive to higher annual growth rates of output.

2 On the Transitions of East Germany and Poland

This section presents data on output growth, and the capital income share, as well as the two policies: transfers from the rest of the world and wage increases, for East Germany and Poland.

After two years of recessions ending in 1991, output per working-age person (i.e. population 15-64) in East Germany and Poland displayed positive growth. Table 1 presents the growth in gross domestic product per working-age person expressed in dollars at purchasing power parities. Between 1989 and 1991, output per working-age person in East Germany declined by 33 percent, while it declined by only 17 percent in Poland. By 2005, output per working-age person grew by more than 60 percent relative to its 1991 level in both economies.

Table 1: Growth in Gross Domestic Product per Working-Age Person (percent).

	$1989-1991$	$1991-2005$
East Germany	-33	66
Poland	-17	63

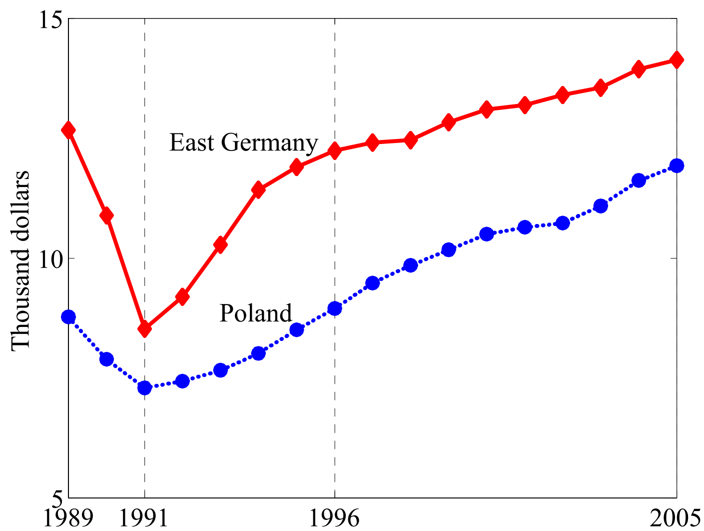
Despite the similarities in total output growth observed over the 15 year period: $1991-2005$, the post 1991 growth experience of East Germany and Poland has been quite different (see Figure 1). East Germany started its transition with very rapid growth. Annual growth in output per working-age person averaged an astounding 7.6 percent, between 1991 and 1996. During the decade that followed, growth slowed down considerably to an average of 1.6 percent per year. Poland's transition started with slower growth: on average 4.2 percent per year from 1991 to 1996. Growth in output per working-age person was sustained at a slightly lower rate in the years that followed: it averaged 3.3 percent per year between 1996 and 2005.

An even more interesting difference between the two economies is in the distribution of income between capital and labor. Over the period 1991 to 2003, the capital income share[5] in Poland was roughly constant, averaging 33 percent (See Figure 2). In East Germany, the capital income share increased dramatically over the same period. In 1991, 12 percent of

[5] As is standard in the literature (see Kravis (1959) and Gollin (2002)), I compute the capital income share

income was attributed to capital. Over a period of 5 years this share more than doubled to a value of 26 percent. By 2003, the capital income share was 30 percent.

Figure 1: Gross Domestic Product per Working-Age Person



For both economies, the period of transition has been eased by transfers received from the rest of the world. Poland benefited from transfers from the European Union. East Germany benefited from transfers from the European Union, and from West Germany. The magnitude of these transfers[6] has been quite different. Prior to accession to the European Union net transfers to Poland amounted to less than 1 percent of GDP. Following the EU accession in May 2004, transfers slightly increased to 1 or 1.5 percent of GDP. Compared to Poland, East Germany received huge transfers. In 1991, net transfers received by East Germany were 52 percent of GDP. Although there has been a significant decline over time, by 2002 transfers still amounted to 30 percent of GDP. The contributions from the European Union were about $3-4$ percent of total transfers.

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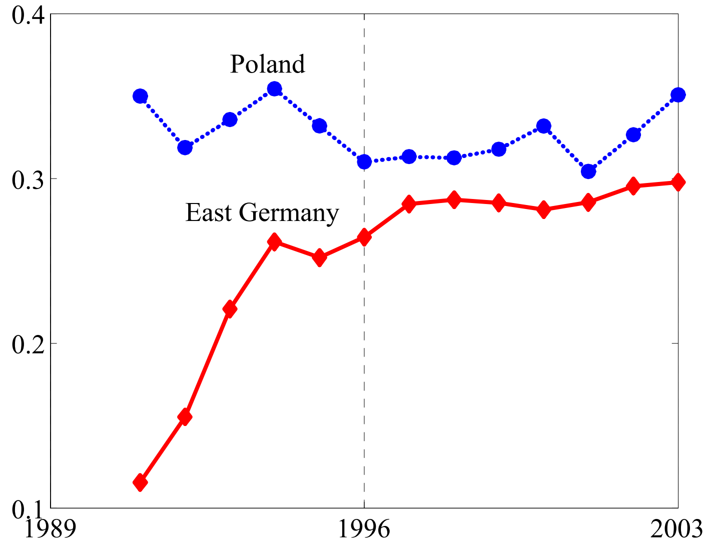
for Poland and East Germany as follows:

$$\zeta = 1 - \frac{\text{Compensation of Employees}}{\text{Total Income-Taxes on Production and Import-Proprietor's Income}}$$

I use GDP as the measure of total income. In subtracting the taxes and proprietor's income from GDP, the implicit assumption is that they are distributed between capital and labor income according to share ζ.

[6] Transfers to Poland and East Germany are reported in studies by the OECD and the European Commission. Transfers to East Germany of similar magnitudes are also reported by other authors. See for example, Ross (2001). Section 7.5 provides details on data sources.

Figure 2: Capital Income Share of Gross Domestic Product



A striking difference between the transitions of East Germany and Poland was in the evolution of wages. Real hourly wage rates increased dramatically in East Germany over the period 1989 – 1991 compared to Poland. Thereafter the growth in real hourly wage rates was comparable[7] (see Table 2). The increases observed for East Germany, during a period of output decreases, indicate wage increases above labor productivity (in fact labor productivity declined during 1989 – 1991). As documented by many authors[8], this was made possible through the intervention of the powerful West German unions and the government's support. Several reasons motivated wage increases, among which are restraining labor from migrating to West Germany, and preventing firms in East Germany from undercutting the West German wage levels. Real wages showed a large increase in Poland as well, between 1991 and 1992. Over the same period, labor productivity, measured as real gross domestic product per employee hour has increased by only 6 percent in Poland. Following 1991 for East Germany and 1992 for Poland, wage growth in both economies did not exceed labor

[7]For East Germany, Krueger and Pischke (1995) report real monthly wage increases for period 1989 – 1990, and Hunt (1999) reports real hourly wage increases for period 1990 – 1991. For Poland, real wage increases over the same period are as reported by Blanchard, Commander and Corricelli (1995). Data for the period 1991 – 2003 for both economies is calculated using national accounts data and total hours worked. See 7.5 for details on data sources.

[8]See for example Hunt (1999), Sinn and Sinn (1992) and Dornbusch and Wolf (1994).

productivity growth.

Table 2: Real Hourly Wage Growth (percent changes).

	East Germany	Poland
1989 – 1990	12.5*	negative
1990 – 1991	33	3*
1991 – 1992	16	18
Average annual growth over 1992 – 2003		
1992 – 2003	2.6	3

*real monthly wages

This section summarized important differences in the transitions of Poland and East Germany. The facts motivate the theory presented in the next section.

3 Theory

In this section, I describe the model economy and characterize the equilibrium.

3.1 Model Economy

The commodities traded at a given point in time are a consumption-capital good and a continuum of differentiated types of labor.

Production Technology

The production technology is modeled in the spirit of Hansen and Prescott (2005). Aggregate output is produced from two factors of production: aggregate labor and capital. The aggregate labor input is a composite of differentiated types of labor. The capital input consists of plants where production can take place.[9]

There are two types of plants distinguished by their productivity level. Let $i \in \{L, H\}$ index the type of plants; L denotes a low productivity plant and H denotes a high productivity

[9] I interpret the plants as representing the whole capital stock of the economy. Equipment represents a small fraction of total capital stock for a given economy. Hence, I consider it to be tied to the structures, and do not model it as a separate stock of capital.

plant. The output of a plant of type i, y_i is given by:

$$y_i = \begin{cases} z_i n_i^{1-\theta} & \text{if } n_i \geq \bar{n} \\ 0 & \text{otherwise} \end{cases}$$

where z_i is the type specific productivity level (with $z_L < z_H$) and n_i is the quantity of composite labor input[10] employed per plant of type i. There is a minimum requirement of (composite) labor necessary to operate a plant. This minimum is given by $\bar{n}$, and is identical across types.

I constrain $\theta \in (0, 1)$. This assumption guarantees that it is optimal to operate many small plants rather than one large plant. Moreover, all operating plants of the same type will employ the same amount of labor. Labor can be moved across locations at no cost, hence it will only be allocated to plants with $n_i \geq \bar{n}$. The requirement that $n_i \geq \bar{n}$ together with a limited labor supply implies an upper bound on the number of plants that can be operated. As a result, at any point in time some plants may be left idle.

Let M_i denote the measure of plants of type i that can potentially be operated and, let N denote the aggregate labor to be employed in a certain period of time. Then, the aggregate production function of this economy is defined by (1), where m_i is the measure of plants of type i that are operated, and are allocated labor input n_i.

$$F(N, M_H, M_L) \equiv \max_{\{m_i, n_i\}_{i \in \{H,L\}}} m_H z_H n_H^{1-\theta} + m_L z_L n_L^{1-\theta} \tag{1}$$
$$\text{s.t. } m_H n_H + m_L n_L \leq N$$
$$0 \leq m_i \leq M_i, \text{ for all } i$$
$$n_i \geq \bar{n}, \text{ for all } i$$

This is a static maximization problem, hence I ignore time subscripts. Finding the maximum output that can be produced in a given period involves allocating the aggregate labor input, N, across the available plants in the economy. Of course, if the number of high

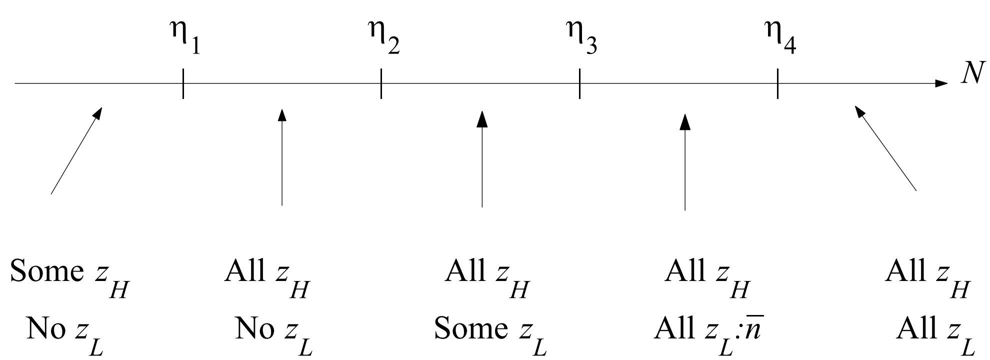
[10] For now, I only consider the choice of the composite plant labor input n_i. The plant labor input, n_i, is a composite of differentiated types of labor, call them $n_i(j)$, with $j \in [0, 1]$. The choice of the differentiated types of plant labor input, $n_i(j)$ can be separated from the choice of the composite plant labor input n_i.

productivity plants, M_H is large enough to employ all the labor that needs to be hired, then it is optimal to leave all the low productivity plants idle. However, if there is a scarcity of z_H type plants some of the z_L type plants will also be operated.

Specifically, in (1) it is clear that the market clearing condition for aggregate labor will hold with equality. However, the inequality constraints on the measures of plants to be operated, m_i, and the inequality constraints on the labor inputs, n_i, either bind or not. For example, maximizing the output produced may involve operating all the plants (i.e. $m_i = M_i$, for all i), while allocating more labor input to more productive plants (i.e. $n_H > \bar{n}$ and $n_L = \bar{n}$). The binding pattern of the inequality constraints will define different branches of the production function. There are five cases that can arrise. To ease their exposition, I consider the stocks of plants M_H and M_L to be fixed and I let the aggregate labor input, N vary. I describe the decision to operate the plants based on the size of N.

Let $\eta_i, i \in \{1, 2, 3, 4\}$ be cutoff values for N. These cutoffs are functions of the stocks M_H and M_L (as well as parameters) and are useful in describing the branches of the production function. Proposition 1. details on the specific form of these endogenous cutoffs. For the time being their particular functional form is not important.

Figure 3: Plants Operated for a Given Aggregate Labor, N



As illustrated in Figure 3, if N is large (i.e. $N \geq \eta_4$), then all high and all low productivity plants are operated (i.e. $m_i = M_i$ for $i \in \{L, H\}$). Given a large labor to be employed, each plant will be allocated more than the minimum labor requirement (i.e. $n_i > \bar{n}$ for $i \in \{L, H\}$). As the size of N declines (i.e. for values of N such that $\eta_4 \geq N \geq \eta_1$)

all the high productivity plants are operated and are allocated at least the minimum labor requirement, $n_H \geq \bar{n}$. However, the low productivity plants employ a smaller and smaller share of aggregate labor. First, all low productivity plants operate, but downsize to $\bar{n}$. Then, only some of these plants operate, $m_L < M_L$, and employ $n_L = \bar{n}$. Finally, all low productivity plants stop operating, $m_L = 0$. For very small values of N, (i.e. $N \leq \eta_1$), only some of the high productivity plants are operated, and are allocated the minimum labor input.

Solving the program (1) for the five cases described above leads to the following aggregate production function.

PROPOSITION 1. Let $\alpha \equiv ((1-\theta) z_H/z_L)^{1/\theta}$ and $\rho \equiv (z_H/z_L)^{1/\theta}$. The aggregate production function is given by:

$$F(N, M_H, M_L) = \begin{cases} z_H \bar{n}^{-\theta} N & \text{if } N \leq \eta_1 \\ z_H M_H^{\theta} N^{1-\theta} & \text{if } \eta_1 \leq N \leq \eta_2 \\ A M_H + z_L \bar{n}^{-\theta} N & \text{if } \eta_2 \leq N \leq \eta_3 \\ z_H M_H^{\theta} (N - M_L \bar{n})^{1-\theta} + z_L \bar{n}^{1-\theta} M_L & \text{if } \eta_3 \leq N \leq \eta_4 \\ z_L (\rho M_H + M_L)^{\theta} N^{1-\theta} & \text{if } N \geq \eta_4 \end{cases} \tag{2}$$

where $\eta_1 = M_H \bar{n}$, $\eta_4 = (\rho M_H + M_L) \bar{n}$ and A, η_2, η_3 are as below:

$$\begin{aligned} \text{If } \alpha &> 1, \text{then } A = \left(z_H \alpha^{1-\theta} - z_L \alpha\right) \bar{n}^{1-\theta}, \eta_2 = \alpha M_H \bar{n}, \text{and } \eta_3 = (\alpha M_H + M_L) \bar{n}, \text{ or} \\ \text{If } \alpha &\leq 1, \text{then } A = (z_H - z_L) \bar{n}^{1-\theta}, \eta_2 = M_H \bar{n}, \text{and } \eta_3 = (M_H + M_L) \bar{n}. \end{aligned}$$

Proof. See Appendix 7.1. ■

The aggregate production function described above has the following properties.

REMARK 1. The aggregate production function, $F : \mathbb{R}^3 \to \mathbb{R}$ is: (i) continuous, (ii) homogenous of degree one, (iii) weakly increasing (iv) differentiable everywhere except at $N = \eta_1 = M_H \bar{n}$, and (v) weakly concave.

At each time t, the aggregate labor input N_t in this economy is a composite of differentiated types of labor. Let $l_t(j)$ denote the labor of type j, $j \in [0,1]$. Then,

$$N_t = \left[\int l_t(j)^\nu \, dj \right]^{1/\nu} \tag{3}$$

where $\nu \in (0,1]$, and $1/(1-\nu)$ represents the elasticity of substitution between the differentiated types of labor.

Given the production technology and (3), at each time, t, the problem of the representative firm can be stated in two parts. First, given factor prices: $r_{H,t}$, $r_{L,t}$ and w_t the firm chooses N_t, $M_{H,t}$, and $M_{L,t}$ to maximize profits:

$$\max_{Y_t, N_t, M_{H,t}, M_{L,t}} Y_t - w_t N_t - r_{H,t} M_{H,t} - r_{L,t} M_{L,t}$$
$$\text{s.t. } Y_t = F(N_t, M_{H,t}, M_{L,t})$$

where $F(N_t, M_{H,t}, M_{L,t})$ is given in (2).

Second, for any given amount of aggregate labor N_t, the demand for each differentiated type of labor is the solution to:

$$\begin{aligned} w_t N_t &= \min_{\{l_t(j)\}, j \in [0,1]} \int w_t(j)\, l_t(j)\, dj \\ \text{s.t. } N_t &\geq \left[\int l_t(j)^\nu \, dj \right]^{1/\nu} \end{aligned} \tag{4}$$

where $w_t(j)$ is the wage for labor of type j.

The demand for labor of type j is given by[11]:

$$l_t(j) = \left(\frac{w_t}{w_t(j)} \right)^{1/(1-\nu)} N_t \tag{5}$$

[11] Similarly, given the composite labor input, $n_{i,t}$ hired by a plant of type z_i, one can derive the demand for each differentiated type of plant labor input $n_{i,t}(j)$. The following holds: $n_{i,t}(j) = (n_{i,t}/N_t) \cdot l_t(j)$, for all $j \in [0,1]$, for all $i \in \{L, H\}$.

where the aggregate wage is $w_t = \left[\int w_t(j)^{\nu/(\nu-1)} dj\right]^{(\nu-1)/\nu}$.

Consumers

There is a large number of infinitely lived consumers with a specific type of labor. The consumers are thought of as being organized in a continuum of unions indexed by j, $j \in [0,1]$. Each union represents all consumers with a specific type of labor. Unions are modeled in the spirit of Blanchard and Kiyotaki (1987). Unions set the wage for labor of type j, and face a downward sloping demand for this labor, as given by (5).

The preferences of a representative consumer in the jth union are given by:

$$\sum_{t=0}^{\infty} \beta^t U\left[C_t(j), l_t(j)\right] \tag{6}$$

The jth union chooses consumption, $C(j)$, investments in the high and low type capital, $X_H(j)$ and $X_L(j)$, rents capital stocks $M_H(j)$ and $M_L(j)$, and chooses the wage rate $w(j)$ to maximize (6) subject to the demand for labor given in (5), the budget constraints,

$$\begin{aligned}(1+\tau_{C,t}) C_t(j) + X_{H,t}(j) + X_{L,t}(j) \leq\ & (1-\tau_{N,t}) w_t(j) l_t(j) \\ & + (1-\tau_{M,t})\left[r_{H,t} M_{H,t}(j) + r_{L,t} M_{L,t}(j)\right] \\ & + \tau_{M,t}\left[\delta_H M_{H,t}(j) + \delta_L M_{L,t}(j)\right] + T_t\end{aligned}$$

as well as the laws of motion for capital:

$$M_{H,t+1}(j) = (1-\delta_H) M_{H,t}(j) + X_{H,t}(j)$$
$$M_{L,t+1}(j) = (1-\delta_L) M_{L,t}(j) + X_{L,t}(j)$$

The union takes the aggregate prices $\{w_t, r_{H,t}, r_{L,t}\}$ and the policies $\{\tau_{C,t}, \tau_{N,t}, \tau_{M,t}, T_t\}$ as exogenously given.

Government

The government taxes consumption at rate $\tau_{C,t}$, labor income at rate $\tau_{N,t}$ and capital income at rate $\tau_{M,t}$. It permits depreciation allowances as given by $\tau_{M,t}\left(\delta_H M_{H,t}(j)+\delta_L M_{L,t}(j)\right)$ for every j. An additional source of government revenues are transfers from the rest of the world. Let these transfers be denoted by $\overline{Tr}_t$. The revenues collected by the government are lump-sum rebated to the households. Let this rebate be denoted by T_t. The government balances its budget every period, hence:

$$\begin{aligned} T_t &= \int \left[\tau_{C,t} C_t(j) + \tau_{N,t} w_t(j)\, l_t(j)\right] dj \\ &+ \int \left[\tau_{M,t}\left(r_{H,t}-\delta_H\right) M_{H,t}(j) + \tau_{M,t}\left(r_{L,t}-\delta_L\right) M_{L,t}(j)\right] dj + \overline{Tr}_t \end{aligned} \tag{7}$$

All the elements of the model have been outlined. I now define an equilibrium.

Definition 1. An equilibrium are allocations $\left\{Y_t, N_t, M_{H,t}, M_{L,t}, \left\{l_t^d(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ and $\left\{C_t(j), l_t^s(j), X_{H,t}(j), X_{L,t}(j), M_{H,t}(j), M_{L,t}(j)\right\}_{t=0}^{\infty}$ for every $j\in[0,1]$, and prices $\left\{r_{H,t}, r_{L,t}, w_t, \left\{w_t(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ such that:

1. Given $\{r_{H,t}, r_{L,t}, w_t\}_{t=0}^{\infty}$, for every $j\in[0,1]$, $w_t(j)$ and $\left\{C_t(j), l_t^s(j), X_{H,t}(j), X_{L,t}(j), M_{H,t}(j), M_{L,t}(j)\right\}_{t=0}^{\infty}$ solve the jth union's problem.
2. Given the prices, $\left\{Y_t, N_t, M_{H,t}, M_{L,t}, \left\{l_t^d(j)\right\}_{j\in[0,1]}\right\}_{t=0}^{\infty}$ solves the firm's problem.
3. The resource constraints hold for all t :

$$l_t^d(j) = l_t^s(j) = l_t(j) \text{ for all } j$$

$$\int M_{H,t}(j)\, dj = M_{H,t}$$

$$\int M_{L,t}(j)\, dj = M_{L,t}$$

$$\int \left[C_t(j) + X_{H,t}(j) + X_{L,t}(j)\right] dj \le Y_t + \overline{Tr}_t$$

3.2 Characterization of the Equilibrium

Before I characterize the equilibrium of this economy, I emphasize a model result.

PROPOSITION 2. The marginal product of the high productivity plants is weakly higher than the marginal product of the low productivity plants. In the case when all z_H type plants are operated, their marginal product is stricly higher than that of the z_L type plants.

Proof. This follows from differentiating F with respect to M_H and M_L. See Appendix 7.1 for details. ■

The immediate implication of Proposition 2. is that there will be no investments in the stock of low productivity plants. The stock of these plants depreciates at rate $\delta_L \geq 0$. Hence, the interesting dynamics of this economy come from the accumulation of the stock of new plants.

The first order conditions of the jth union problem are summarized by the budget constraints, the laws of motion for capital, $X_{L,t}(j) = 0$, as well as:

$$\begin{aligned}
\frac{U_C\left(C_t\left(j\right), l_t\left(j\right)\right)}{U_C\left(C_{t+1}\left(j\right), l_{t+1}\left(j\right)\right)} &= \beta \frac{1+\tau_{C,t}}{1+\tau_{C,t+1}} \left(1 + \left(1-\tau_{M,t+1}\right)\left(r_{H,t+1} - \delta_H\right)\right) \\
w_t\left(j\right) &= -\frac{1}{\nu} \frac{\left(1+\tau_{C,t}\right)}{\left(1-\tau_{N,t}\right)} \frac{U_l\left[C_t\left(j\right), l_t\left(j\right)\right]}{U_C\left[C_t\left(j\right), l_t\left(j\right)\right]} \\
\lim_{t\to\infty} \beta^t U_C\left[C_t\left(j\right), l_t\left(j\right)\right] M_{H,t+1}\left(j\right) &= 0
\end{aligned} \tag{8}$$

where the current period utility is $U(C,l) = \log(C) + \psi \log(1-l)$.

The first order conditions of the representative firm's problem are:

$$r_{i,t} = \frac{\partial F\left(N_t, M_{H,t}, M_{L,t}\right)}{\partial M_{i,t}}, \; i \in \{L, H\}, \qquad w_t = \frac{\partial F\left(N_t, M_{H,t}, M_{L,t}\right)}{\partial N_t}$$

Given the symmetry of the unions, they all make the same choices. In particular, $w_t(j) = w_t$ and $l_t(j) = N_t$. Thus, from now on I drop the j subscripts.

The factor $1/\nu$ present in the intratemporal condition (8) represents the markup of wages above their competitive levels. If $\nu = 1$, wages are competitive, i.e. unions have no power. However, if $\nu < 1$ (i.e. $\nu \in (0,1]$), unions have power, and wages will be above wages in a competitive economy. In numerical experiments, I consider union power to vary over time, hence I use the notation $1/\nu_t$ to denote the markup over competitive wages at date t.

Next, I analyze the long-run behavior of the model economy.

PROPOSITION 3. As the stock of low productivity plants approaches zero asymptotically, the aggregate production function of the economy becomes:

$$\lim_{M_L \to 0} F(N, M_H, M_L) \equiv F(N, M_H) = \begin{cases} z_H \bar{n}^{-\theta} N & \text{if } N \leq M_H \bar{n} \\ \\ z_H M_H^{\theta} N^{1-\theta} & \text{if } N \geq M_H \bar{n} \end{cases} \tag{9}$$

Proof. As $M_L \to 0$, the aggregate production function is the solution to:

$$F(N, M_H) \equiv \max_{m_H, n_H} m_H z_H n_H^{1-\theta} \tag{10}$$

$$\text{s.t. } m_H n_H \leq N$$

$$m_H \leq M_H$$

$$n_H \geq \bar{n}$$

The solution to (10) is incorporated in the proof of proposition 1. See Appendix 7.1 for details. ■

The production function in (9) can be understood as follows. In the case in which $N \geq M_H \bar{n}$, all high productivity plants are operated and are allocated at least $\bar{n}$ units of labor. However, whenever $N < M_H \bar{n}$, the aggregate labor employed is insufficient for all high productivity plants to be operated. Therefore, some of them will be left idle. Since z_H type plants are not a scarce input into production they will earn a share of total income equal to 0; the labor share of total income in this case is 1.

PROPOSITION 4. The steady state of this economy is such that all z_H type plants operate $N \geq M_H \bar{n}$. Thus, the production function in the steady state is given by $F(N, M_H) = z_H M_H^{\theta} N^{1-\theta}$.

Proof. Suppose there is a steady state such that $N < M_H\bar{n}$. Given that the return to investing in M_H is zero (i.e. $r_H = 0$) there will be no investments undertaken: $X_H = 0$. Thus, the capital stock M_H depreciates. This contradicts the fact that in a steady state M_H is constant. This completes the proof. ■

To summarize: In the steady state of this economy only high productivity plants operate. Furthermore, *all* high productivity plants operate.

Now let's consider an economy that transitions from a low productivity technology, z_L, to a high productivity technology, z_H. Given an initial low measure of high productivity plants, the transition begins with all plants (i.e. low and high productivity plants) operating. As the stock of high productivity plants increases over time, first, all low productivity plants operate at the minimum scale, $\bar{n}$, then some of them become idle. Finally, only new plants are operated. These stages of transition generate interesting dynamics of factor income shares.

4 Implications for Output Growth and Factor Income Shares

In this section, I describe the model's implications for factor income shares and output growth. I shut down different elements of the model presented in Section 3. in order to isolate their contribution to model outcomes. First, I analyze an economy with no transfers from the rest of the world and no union power. I refer to this economy as the **benchmark economy**. Second, I add transfers from the rest of the world to the benchmark economy. Then, I add union power to the benchmark economy. Finally, I analyze in more detail the effect of both policies on the factor income shares.

4.1 Benchmark Economy

Let a benchmark economy be an economy as described in Section 3, with $\overline{Tr}_t = 0$ and $\nu_t = 1$ (i.e. no union power) for all t.

Output Growth

An economy that transitions to a higher productivity technology, starts out with a high fraction of z_L type plants and a low fraction of z_H type plants. Consumers invest in the better technology. Hence, z_H type plants accumulate fast, and there is fast growth in output from the beginning of the transition.

Factor Income Shares

The labor income share is defined as the share of labor income in total income (i.e. wN/Y). The capital income share is defined as the share of gross capital income in total income (i.e. $(r_H M_H + r_L M_L)/Y$). I focus the exposition on the capital income share and present its dynamics. First, I present the plant level capital income share (or profit share) for both plant types; then I relate the plant profit shares to the capital income share for the whole economy.

Let π_i and ϕ_i denote the total profits and the profit share of a plant of type i, respectively. Then $\phi_i = \pi_i/y_i$.

PROPOSITION 5. If $\alpha > 1$ and all high productivity plants operate at any given point in time, the profit share at the plant level[12]:

(i) is constant for high productivity plants, i.e. $\phi_H = \theta$, and

(ii) varies for low productivity plants, i.e. $\phi_L \leq \theta$, with equality if $n_L > \bar{n}$.

Proof. See Appendix 7.1. ■

In this economy, the labor productivity is weakly higher at the high productivity plants compared to the low productivity plants; and strictly higher whenever z_L type plants operate at $\bar{n}$. Moreover, if $\alpha > 1$, labor productivity at the z_H type plant is proportional to the wage rate (i.e. equals $w/(1-\theta)$). This yields a constant profit share at these plants. In contrast, the profit share at the low productivity plants varies; it is lower than θ whenever $n_L = \bar{n}$.

The result of Proposition 5. is important since it helps in understanding the behavior of the capital income share in this economy. Let ϕ denote the economy wide capital income share. Then:

$$\phi = \frac{\Pi_H + \Pi_L}{Y} \tag{11}$$

[12] The requirement $\alpha > 1$ is equivalent to $z_L/z_H < 1 - \theta$. Given a value for θ of 0.33, this is equivalent to a high productivity technology that is roughly 50 percent more productive than the low productivity technology. I find $\alpha > 1$ to be consistent with data observations (see Section 5. for details). Hence, I focus exclusively on this case.

where Π_i are total profits of plants of type i, and Y is the aggregate output.

Let Y_i denote the total output produced by plants of type i. Then, (11) becomes:

$$\phi = \frac{\Pi_H}{Y_H}\frac{Y_H}{Y} + \frac{\Pi_L}{Y_L}\frac{Y_L}{Y}$$

Notice that:

$$\frac{\Pi_i}{Y_i} = \frac{\pi_i M_i}{y_i M_i} = \frac{\pi_i}{y_i} = \phi_i$$

Making use of Proposition 5., ϕ can be written as:

$$\begin{aligned}
\phi &= \phi_H \frac{Y_H}{Y} + \phi_L \frac{Y_L}{Y} \\
&= \theta \frac{Y_H}{Y} + (\phi_L + \theta - \theta) \frac{Y_L}{Y} \\
&= \theta \left(\frac{Y_H}{Y} + \frac{Y_L}{Y} \right) - (\theta - \phi_L) \frac{Y_L}{Y} \\
\phi &= \theta - (\theta - \phi_L) \frac{Y_L}{Y} \qquad (12)
\end{aligned}$$

There are a few things to learn from (12). If the profit share of z_L type plants is θ (i.e. $\phi_L = \theta$), or the total output produced by these plants is zero, the economy's capital income share, ϕ equals θ. Hence, deviations of ϕ from θ are possible if and only if $\phi_L < \theta$ and $Y_L > 0$. In this situation, $\phi < \theta$ given that the second term in (12) is positive.

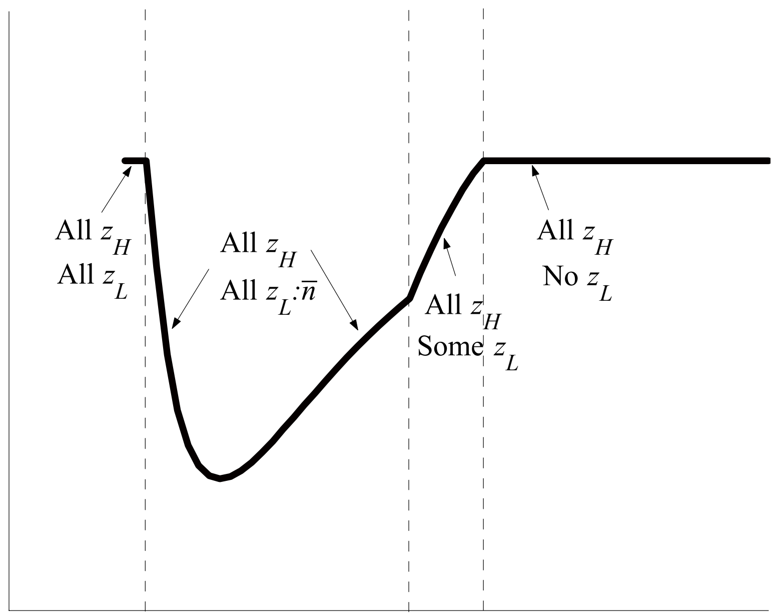
The low productivity plants have a profit share lower than θ whenever they hire the minimum labor requirement, $\bar{n}$ (Proposition 5). Hence, the capital income share of this economy, ϕ is different from θ in two cases: (i) all low productivity plants operate and they hire $\bar{n}$ labor, or (ii) only some low productivity plants operate and hire $\bar{n}$ labor.[13]

Let's consider again an economy that transitions from a low productivity technology, z_L to a high productivity technoogy, z_H. Initially, all plants will operate and hire labor $n_i > \bar{n}$. Thus, the economy starts out with $\phi = \theta$. As the stock of high productivity plants increases over time, the low productivity plants first operate at $\bar{n}$, then they stop operating. During this process, the capital income share falls below θ. In the steady state of the economy, the

[13] Theoretically, the capital income share ϕ differs from θ also when some high productivity plants are idle. Since idleness of the z_H type plants does not occur in equilibrium (see Proposition 4.), I do not focus on this case.

capital share is constant again: $\phi = \theta$. Figure 4 presents a generic capital income share in this economy. The U-shaped dynamics of the capital share are a robust result in this economy.

Figure 4: Capital Income Share in a Transition to a Better Technology



4.2 An Economy with Transfers from the Rest of the World

I now consider adding exogenous transfers from the rest of the world to the benchmark economy. I refer to this economy as an economy with transfers.

Suppose there are no transfers at time $t = 0$, and there is a permanent increase in the flow of transfers starting at time $t = 1$, $\overline{Tr}_t = \overline{Tr}_1$ for all $t \geq 1$. In order to understand the impact of transfers on output growth and the capital income share, I first describe the impact on hours worked. In the economy with transfers, hours worked, N, will be lower compared to the benchmark economy. Hours are lower since the household is now richer. Moreover, N will experience a drop at time 1 with the introduction of the policy. Of course, these effects on hours worked will be stronger the higher the size of the transfers.

Let's consider the effect transfers have on output growth. If transfers are large enough, the drop in hours worked will be associated with a negative growth in output from time $t = 0$ to $t = 1$. However, following period 1 the stock of capital, M_H will accumulate faster in the economy with transfers compared to the benchmark, thus leading to higher output growth.

What is the effect of this policy on the capital income share? Similar to the benchmark economy, the capital income share in the economy with transfers has a U-shape (see Figure 4). Compared to the benchmark, the decline in the capital share may occur sooner and may be of a larger magnitude. Given lower hours worked in an economy with transfers, each plant's labor input n_i will be lower. If transfers are large enough, they trigger low productivity plants to operate at scale $\bar{n}$ sooner than in the benchmark model. Hence, as described in Section 4.1. the economy's capital share declines below θ, sooner. To understand why the decline in the capital share may be of larger magnitude in the economy with transfers, recall that the capital share ϕ equals:

$$\phi = \theta - (\theta - \phi_L) \frac{Y_L}{Y}.$$

Given z_L type plants operate at $\bar{n}$, their profit share ϕ_L is lower than θ, and moreover is lower than in the benchmark model. This is due to a higher equilibrium wage rate in the economy with transfers.[14] Moreover, when all z_L type plants operate, the share of total output produced by low productivity plants is higher in an economy with transfers compared to the benchmark. The reason behind this is that (i) the z_L type plants have a fixed output[15], given by $z_L \bar{n}^{1-\theta}$, while (ii) z_H type plants decrease their labor input in response to declines in N, and hence their output decreases. To summarize, a lower ϕ_L and a higher Y_L/Y yield a lower capital income share, ϕ, in an economy with transfers compared to the benchmark economy.

4.3 An Economy with Unions

I now consider adding unions to the benchmark economy. I refer to this economy as the unionized economy.

Suppose unions have no power at time $t = 0$ (i.e. $\nu_0 = 1$) and there is a permanent increase in union power thereafter. That is, $\nu_t < 1$ for all $t \geq 1$. Of interest is the impact of union power on output growth and the capital income share.

[14] When z_L operate at $\bar{n}$, profits are $\pi_L = z_L \bar{n}^{1-\theta} - w\bar{n}$. Profit share is $\phi_L = 1 - w / \left(z_L \bar{n}^{-\theta}\right)$. A higher wage rate in an economy with transfers, thus yields a lower profit share compared to the benchmark economy.

[15] Given a low stock of high productivity plants during the beginning of the transition, the low productivity plants cannot be shut down. Thus, they are operated at the minimum labor requirement.

The effect of union power on output growth will depend on the specified process for ν_t. If ν_t is constant for all $t \geq 1$, the unionized economy will have lower growth rates of output during the transition to the steady state compared to a competitive economy. Both hours and accumulation of capital will be lower in this economy compared to the benchmark economy. The effect of union power on the capital income share is similar to the effect of transfers. Capital income share in the economy with unions displays a U-shape similar to the benchmark economy (see Figure 4). Compared to the benchmark, this decline in the capital share may occur sooner and may be of a larger magnitude. With the introduction of union power, the wage rate increases and the hours worked, N, decrease. The effects on the capital share are motivated by similar mechanisms as described in Section 4.2.

4.4 A Closer Look at Implications for Factor Income Shares

Here, I consider in more detail the effects of the two policies: transfers and union power on the capital income share. The purpose of this section is to point out that policies may affect the capital income share in a nonlinear fashion.

Suppose at time 0, there are no transfers and no union power, i.e. $\overline{Tr}_0 = 0, \nu_0 = 1$. I consider the following policies: (i) A permanent increase in the transfer starting period 1, i.e. $\overline{Tr}_t = \overline{Tr}$ for all $t \geq 1$. The transfer is expressed as a percentage of GDP. (ii) A permanent increase in union power starting period 1, i.e. $1/\nu_t = 1/\nu$ for all $t \geq 1$, where $\nu < 1$. The factor $1/\nu$ determines the markup of wages above the competitive wage.

The experiments performed are presented in Table 3. First, I consider the model economy with a given policy. I increase the magnitude of that policy in a linear fashion and examine how the capital income share in a particular time period of the model varies. Second, I consider the interaction between the two policies. For each experiment, I compute equilibrium paths for every value of the policy. For example, in experiment 1, I compute equilibrium paths 61 times, corresponding to the 61 values of the transfers $\overline{Tr}$. To assess the effect of the change in policy on the capital income share, I focus on a snapshot in time. Specifically, I present the capital income share at time $t = 3$, as the policies vary. In Appendix 7.3, I present time series for the capital share in two of the experiments considered here. The parameter values used in the experiments are also presented in Appendix 7.3.

Table 3: Impact of Policies: Four Experiments.

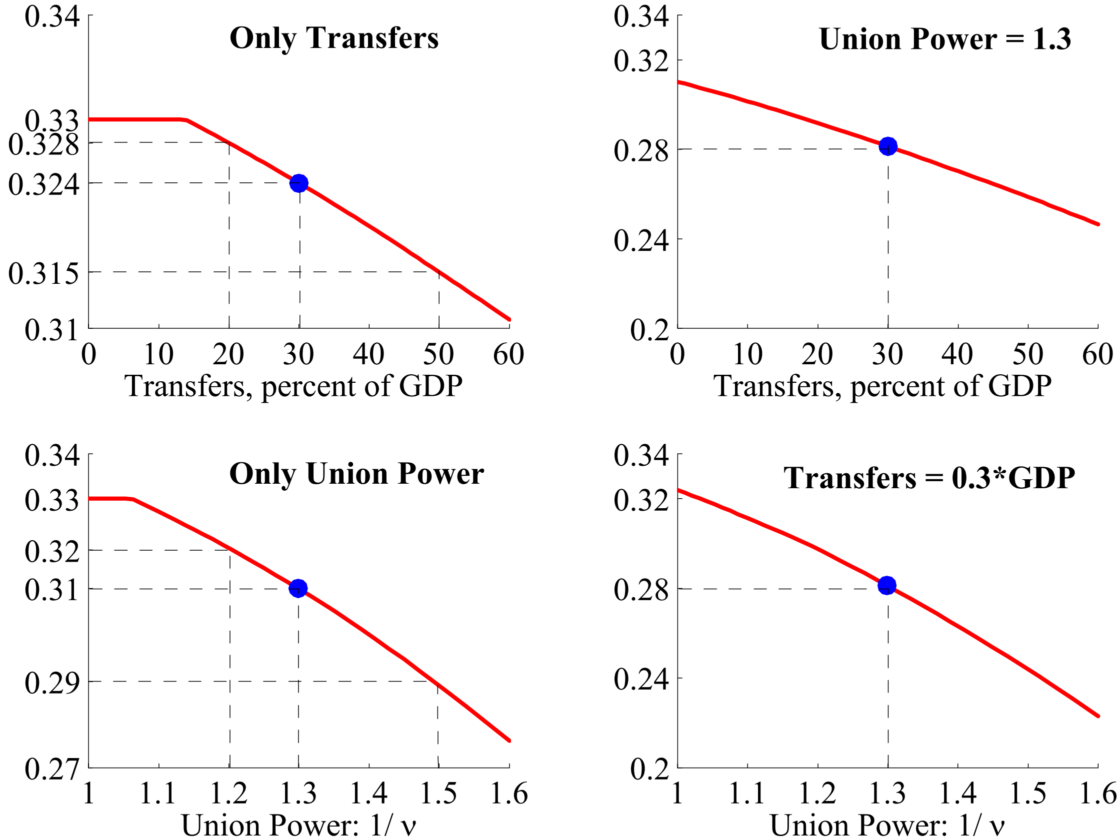
	Transfers, % of GDP	Union power
Experiment 1.	$\overline{Tr} \in [0, 60]$	$1/v = 1$
Experiment 2.	$\overline{Tr} = 0$	$1/v \in [1, 1.6]$
Experiment 3.	$\overline{Tr} \in [0, 60]$	$1/v = 1.3$
Experiment 4.	$\overline{Tr} = 30$	$1/v \in [1, 1.6]$

The results from the four experiments are presented in Figure 5. The top, right-hand panel shows the results from Experiment 1. As is seen in the figure, whenever transfers are a small percentage of GDP, they do not have an impact on the capital income share at time $t = 3$. For transfers of 60% of GDP, the capital income share is approximately 2 percentage points lower than θ. As transfers increase from 0 to 60% of GDP, one can observe a nonlinear impact on the capital income share. Consider transfers of 20, and 30 percent of GDP, respectively. The capital income share at $t = 3$ is roughly 0.2, respectively 0.6 percentage points below 0.33. However, transfers of 50% (= 20% + 30%) of GDP yield a capital income share of 0.315, 1.5 percentage points lower than 0.33. Thus, a linear increase in the policy yields a non-linear change in the capital income share. The same type of result is observed when varying the union power parameter (see Experiment 2).

The nonlinear effects observed are driven by asymmetric responses of the two plant types to policies. As policies increase, the aggregate labor, N decreases. In response to declines in N, high productivity plants always lower their labor input and maintain a capital share of θ. However, low productivity plants are forced to keep labor at $\bar{n}$ for high magnitudes of the policies and thus have a lower than θ profit share. Hence, as policies increase: (*i*) a higher fraction of total output is produced by the low productivity plants and (*ii*) the profit share of these plants declines. These two effects amplify each other (recall $\phi = \theta - (\theta - \phi_L) Y_L/Y$). Thus, ϕ declines in a nonlinear fashion with policy increases.

I now turn to Experiments 3 and 4 and illustrate the interaction between the two policies. I focus on a specific example. By themselves, transfers of 30 percent of GDP yield a drop in the capital income share of 0.6 percentage points. By itself, union power of 1.3

Figure 5: Capital Income Share at time $t = 3$, in Four Experiments



yields a drop of 2 percentage points in the capital income share. However, when considered together, transfers of 30 percent of GDP and union power of 1.3 yield a drop of 4.9 percentage points in the capital share, much larger than $0.6 + 2 = 2.6$. The intuition behind this results is similar to the one provided above. Transfers of 30 percent of GDP alone yield the low productivity plants to operate at $\bar{n}$ and earn a profit share $\phi_L < \theta$. Hence $\phi < \theta$. When introducing the second policy: union power into the model, the low productivity plants still operate at $\bar{n}$ and their profit share drops further.

In the above example, the two policies together yield a 4.9 percentage points fall in the capital income share. A decline of the same magnitude can be obtained with one policy alone if that policy is large enough. For example, by itself, union power $1/\nu = 1.56$ yields a 4.9 percentage point fall in the share. Similarly, a value of transfers higher than 60% of GDP could deliver the same impact on ϕ.

5 Applications of Theory

In this section, I choose model parameters to match key facts of the East German and Polish economy. I then examine the output growth and capital income share of output for the two economies.

5.1 Parameter Choices

I consider the model economy described in Section 3 with no transfers and no union power $\left(\text{i.e. } \overline{Tr}_t = 0 \text{ for all } t \text{ and } \nu_t = 1 \text{ for all } t\right)$ to be the benchmark economy. This model has the following parameters[16]: $\{\beta, \delta_H, \delta_L, \theta, \psi, z_H, z_L, \bar{n}, M_{H,1989}, M_{L,1989}\}$ as well as the exogenous processes: $\{\tau_{C,t}, \tau_{N,t}, \tau_{M,t}, T_t\}$. I choose parameters such that this benchmark model matches certain key facts for the East German economy. I then emphasize the parameters that are common for the two economies or economy specific. Furthermore, I assign values to the economy specific parameters for Poland. I consider the choices for the policy parameters: $\left\{\overline{Tr}_t, 1/\nu_t\right\}$ when discussing the experiments performed for the two economies (see Sections 5.2 and 5.3). These parameters are not present in the benchmark model.

The exogenous processses $\{\tau_{C,t}, \tau_{N,t}, \tau_{M,t}, T_t\}$ are obtained from Mendoza et al. (1994). The tax rates on consumption, labor income and capital income used are averages over a specified period. That is, in the model $\tau_{i,t} = \tau_i$ for all t, for all $i \in \{C, N, M\}$. To obtain tax rates for East Germany, I use estimates for all of Germany since taxation is quite similar between East and West Germany. I use the estimates provided by Mendoza et al. (1994) for Germany, for the period $1991 - 1996$. This yields $\tau_C = 0.16$, $\tau_N = 0.42$, and $\tau_M = 0.26$. Given the tax rates $\tau_{i,t}$ above and the assumption that the government balances its budget every period, the model lump sum transfers, T_t, are determined residually.

Now I determine the parameters $\{\theta, \delta_H, \delta_L, \beta, \psi, M_{H,1989}, M_{L,1989}, z_H, z_L, \bar{n}\}$.

The capital income share, θ, is based on national income accounts data for all of Germany. This yields $\theta = 0.33$. The annual depreciation rate of the capital stocks is considered to be the same for both stocks: M_H and M_L. It is equal to the ratio of consumption of fixed

[16] I identify the first period in the model with year 1989; hence $M_{H,1989}$ and $M_{L,1989}$ indicate the model's initial conditions for capital stocks.

capital to the net stock of fixed assets for Germany, $\delta_H = \delta_L = 0.06$. Next, I use the model's steady state Euler Equation to determine the value of β :

$$\beta = \frac{1}{1 + (1 - \tau_M)\left(\theta \frac{Y}{M_H} - \delta_H\right)}$$

where M_H/Y is the capital output ratio in the steady state of the model and it matches the average capital output ratio for Germany over the period $1991 - 2003$. Given the steady state M_H/Y and parameters given above, the resulting after tax interest rate $(1 - \tau_M)\left(\theta \frac{Y}{M_H} - \delta_H\right)$ is 4.3 percent. The discount factor obtained is $\beta = 0.96$.

The value of ψ is determined such that hours worked, N, in model year 1991 equal the value of 0.21 observed in East German data. The value obtained is $\psi = 1.82$.

The capital stocks are determined as follows: the value of $M_{H,1989} + M_{L,1989}$ is chosen to match a capital-output ratio of 2.3 reported for East Germany for year 1991. Given that the first period in the model is year 1989, and that capital-output ratio increases during the transition to a new steady state, the model will have a capital-output ratio in 1989 slightly lower than 2.3. Concerning the split of the capital stock in high and low productivity capital, I assume that the high productivity capital stock represents 5 percent of the total capital stock in 1989. I choose a low share of high productivity capital to reflect the view that the capital stock of East Germany was outdated (See for example Sinn and Sinn (1992)).

I am left with assigning values for the technology parameters $\{z_H, z_L, \bar{n}\}$. I pick the ratio z_H/z_L such that the model delivers an increase in Total Factor Productivity (TFP) of 17 percent[17] as observed in East German data for the period: 1991 to 2003. Let A_t denote model TFP. I calculate A_t as a Solow residual from a standard Cobb-Douglas production function:

$$A_t = \frac{Y_t}{(M_{H,t} + M_{L,t})^{\theta} N_t^{1-\theta}}$$

where $M_{H,t} + M_{L,t}$ represents the total capital stock of the economy at time t. In Appendix 7.4, I perform a sensitivity analysis for the ratio z_H/z_L.

[17] In East German data, TPF showed a rapid increase between 1991 and 1994. Since 1994, TFP was roughly constant. Using detrended, per working-age person data on output and capital stock, and data on average hours worked, I find that between 1994 and 2003, TFP was on average 17 percent above its 1991 level. I use detrended, per working age person data on output and capital stock to keep model and data comparable.

I determine the level of z_L such that the model matches the level of output per working-age person observed in East Germany for the year 1991. I then use the z_L/z_H ratio from above to obtain the level of z_H. Finally the value of $\bar{n}$ is determined by the steady state of the model. Recall in the steady state, $N \geq M_H \bar{n}$. I pick $\bar{n} \simeq N^*/M_H^*$, where star denotes steady state.

The parameters that are common to East Germany and Poland are displayed in Table 4. The technology parameters z_H/z_L and $\bar{n}$ are common since the two economies are considered to transition to the same technology. Both economies start with capital of type z_L and build new capital that is z_H/z_L more productive. Although the ratio z_H/z_L is the same for both economies, the levels of z_L and z_H will differ for the two economies. This level difference is due to different levels of output per working-age person observed in data (see Figure 1) . I also take the initial fraction of high productivity capital stock in total capital to be the same for the two economies.

Table 4: Common Parameters.

Parameter	Description	Value
β	Discount factor	0.96
$\delta_H = \delta_L$	Depreciation rate for capital	0.06
θ	Capital income share	0.33
ψ	Preference parameter	1.82
z_H/z_L	Capital productivities ratio	1.53
$M_H/(M_H + M_L)$	Fraction of capital stock of type z_H	0.05
$\bar{n}$	Minimum hours requirement	0.005

I obtain economy specific parameters for Poland by using the same method as described above (see Table 5). To obtain tax rates, I use data from the National Income and Product Accounts and Revenue Statistics. The tax rates are constructed for the period $1995 - 2003$, given data availability. The level of z_L is chosen to match the level of output per working-age person in 1991. The estimate for the capital-output ratio in Poland for year 1989 is obtained from Bems (2005) .

Table 5: Economy Specific Parameters.

Parameter	Description	Value for EG	Value for Poland
z_H	High productivity level	11.45	10.25
z_L	Low productivity level	7.48	6.7
$(M_H + M_L)/Y$	Capital-output ratio	2.3*	1.7*
τ_C	Tax rate on consumption	0.16	0.18
τ_N	Tax rate on labor income	0.42	0.38
τ_M	Tax rate on capital income	0.26	0.28

*The capital-output ratios are for year 1991 for East Germany, and 1989 for Poland.

5.2 Application to East Germany

Given the specified parameters, I perform a few experiments to evaluate the model's performance with regard to output growth and the capital income share of output. I consider the results of the benchmark model, as well as three experiments. Recall the benchmark model is an economy with no transfers from the rest of the world and no union power. In experiment 1, I introduce transfers alone to the benchmark model; in experiment 2, I introduce union power alone to the benchmark model. Experiment 3 considers the impact of both transfers and union power.

Before I proceed, I motivate the choice of the policy parameters $\{\overline{Tr}_t, 1/\nu_t\}$.

The transfers, $\overline{Tr}_t$ to East Germany represent net transfers from West Germany as well as the European Union (see Section 2). Transfers are zero in model period 1989. Up to year 2002, I use estimates of total net transfers provided by the European Commission.[18] After year 2002, I let transfers decline over time. This is motivated by the fact that transfers from West Germany are expected to end in 2019 as documented in the Solidarity Pact II. This contractual aggreement signed by the federal states of Germany is aimed at promoting the development of East Germany for the period 2005 to 2019. Hence, transfers is the model are zero starting period 2019.

To measure the extent of union power, I use data on real hourly wage increases for East Germany. In year 1989, there is no union power $(\nu_{1989} = 1)$.[19] I pick the parameter

[18] See Appendix 7.5 for complete data sources.

[19] Alternatively, one could consider a value of $\nu_{1989} \neq 1$. Thereafter increases in union power are relative to

ν_t for model years 1990 and 1991 such that the experiment with both transfers and union power captures the real hourly wage increases observed in data for the period $1989 - 1991$ (See Table 2).[20] Thereafter, I keep union power constant. In Appendix 7.4, I consider the possibility of union power declining over time. A declining union power could be motivated by a decline in the share of union employees to total employees, observed since 1991.

Below, I report model results for the growth rates of output. The first line in Table 6 presents the data. I use gross domestic product per population aged $15-64$ that is detrended with an annual trend growth of 1.012. These transformations make the data comparable to the model, given that the current model does not incorporate balanced growth due to technology or population change.[21] I choose a 1.2 percent annual trend growth, corresponding to the growth rate of output per working-age person in West Germany since 1990. I focus on the performance of the model for the 3 periods of time indicated in Table 6. The reason for focusing on these particular periods is the following: 1991 was the first year following 1989 to see positive growth in output per working-age person in East Germany. Moreover starting 1996 growth in output per working-age person has slowed down considerably; we observe roughly trend growth for the period $1996 - 2004$.

Table 6: East Germany: Growth Rates of Output per Working-Age Person.

	Average annual growth over period (in %)		
	1989 − 1991	1991 − 1996	1996 − 2004
Data*	−18.8	6.3	0.4
Benchmark	8.4	3.8	1.0
Experiment 1: Transfers	−1.1	6.3	2.7
Experiment 2: Union Power	−0.9	2.2	0.9
Experiment 3: Both	−8.5	4.0	2.3

*detrended using a 1.2 percent annual trend.

this number.

[20] For the period 1989 and 1990, Krueger and Pischke report real monthly wage increase of 12.5 percent. The hourly wage increase over the same period was potentially larger, due to declines in hours worked. However, I choose ν_{1990} to match the value of 12.5% reported, given lack of data on hourly wage increases.

[21] I conjecture that presence of balanced growth will not alter the model results presented; hence I abstract from it.

Let's consider the first time period: $1989 - 1991$. Data show an average annual decline in output per working-age person of 19 percent. The benchmark model is not able to capture this drop. Experiments 1 and 2 account for about 5 percent of this drop. However, experiment 4 accounts for 45% of the decline in output per working-age person. Let me explain this result. In all model experiments the aggregate labor, N, declines and capital stock increases between $1989 - 1991$. Hence, the model's decline in output is due to a decline in hours worked.[22] In the benchmark model, the decline in labor is too small to lead to a decline in output. In experiments 2 (union power alone) and 3 (union power and transfers), the decline in labor is of similar magnitudes and is larger than in the benchmark model. However, the growth of the capital stock in these experiments differs from the benchmark. The model with both union power and transfers will have a lower initial growth in the capital stock due to an income effect from the presence of transfers. Thus, the decline in labor supply, N, has a larger impact in experiment 3 compared to experiment 2. Moreover, the model with transfers alone is not able to capture much of the output decline, due to the fact that hours worked decline by less than in models with union power.

Over the period $1991 - 1996$, output per working-age person in the data grew dramatically. On average growth was 6.3 percent above trend. A model with transfers from the rest of the world (experiment 2) captures this fast growth extremely well. The model with both union power and transfers (experiment 3) does pretty well, too: it accounts for 63 percent of the growth over this period. However, for the period $1996 - 2004$ the two models fail to

[22] In the model with transfers and unions, the decline in hours worked, N, between 1989 and 1991 is 6.3 hours per week. Below I show that this magnitude is in line with data. Average hours worked by a person of age $15 - 64$, equal:

$$\frac{H}{\text{Population } 15 - 64} = \frac{H}{E} * \frac{E}{\text{Population } 15 - 64},$$

where H denotes total hours worked and E denotes total employment.

Hunt (1999) reports a 25 percent drop in total employment for East Germany over 1989 to 1991. Dornbusch and Wolf (1994) report a similar number. Moreover, population age $15 - 64$ has decreased by 3 percent over the period. Cociuba (2006) uses the German Socio-Economic Panel and identifies a 1 percent increase in hours worked by employees between 1989 and 1991. Given a very low fraction of self employed in the total population employed, hours per employee are a good proxy for H/E. Thus, I obtain a drop in average hours worked of roughly 22 percent between 1989 and 1991. Given a level of average hours worked in 1991 of 21 hours per week, this implies a drop of 5.9 hours per week between 1989 and 1991. The comparable number for Poland is a decline of 4 hours per week between 1989 and 1991. Poland experienced a smaller drop in average hours since the decline in employment was much lower (i.e. 10 percent between 1989 and 1991 as reported in data provided by the Groningen Growth and Development Center).

capture the very slow growth in output per working-age person observed in data. The current model did not have a chance to capture this slowdown in the growth since it abstracts from factors that yield a slowdown in total factor productivity as observed in the East German data. Cociuba (2006) performs a business cycle accounting exercise as in Chari, Kehoe McGrattan (2006) and shows that understanding the slowdown in productivity growth will help understand the very slow growth in output per capita.

I now turn to the model's predictions for the capital income share of output (see Table 7). Data for the capital share is available starting year[23] 1991. All models display a capital income share, ϕ, of 0.33 for the beginning two years. For the year 1991, the model with transfers alone yields a decline in ϕ of 1 percentage point below 0.33; the model with union power alone yields a decline in ϕ of 5 percentage points. Experiment 3, which considers the two policies at the same time, yields a decline in ϕ of 11 percentage points. This amplification effect was described in Section 4.4 in detail.

Table 7: East Germany: Capital Income Share of Output.

	1991	1996	1997 – 2003
Data	0.12	0.26	0.29
Benchmark	0.33	0.29	0.29
Experiment 1: Transfers	0.32	0.27	0.27
Experiment 2: Union power	0.28	0.26	0.27
Experiment 3: Both	0.22	0.23	0.25

I conclude that a model with union power and transfers is able to capture a very low initial capital income share and a fast initial growth for the East German economy.

[23] For year 1989, all data necessary to compute the capital income share in East Germany are not available. However, available data indicate a low compensation share of output of roughly 40 percent of GDP, and a depreciation of capital of 15 percent of GDP (See Appendix 7.5 for data sources). These facts indicate that capital income share was not lower in 1989 than in 1991. In fact, given the low labor income share in 1989, data seem to indicate that capital income share was quite high.

5.3 Application to Poland

I now perform experiments for Poland. First, I motivate the choice of the policy parameters $\left\{\overline{Tr}_t, 1/\nu_t\right\}$.

The transfers to Poland, $\overline{Tr}_t$ represent net transfers from the European Union (see Section 2). Transfers are zero for the model period 1989, and 2 percent of GDP thereafter. Given the recent accession to the EU, I consider that these transfers will not decline to 0 in the near future, hence steady state transfers are positive, at 2 percent of GDP.

To measure the extent of union power, I apply the same method as for East Germany. I choose ν_t to equal 1 for year 1989 to reflect absence of union power. I then choose ν_t to match the wage increases of 3 and 18 percent from 1990 to 1991, and 1991 to 1992, respectively (I set $\nu = 1$ for model year 1991 as well, since the model will predict higher wage growth than 3 percent for that year. I set $\nu < 1$ for year 1992). Thereafter, I keep union power constant. In Appendix 7.4, I consider the possibility of union power declining over time.

I now consider the performance of the benchmark model and the experiment in which both transfers and union power are introduced. I only present results for these two experiments. Given the small magnitude of the transfers, a model with transfers alone has predictions very similar to a model with no transfers at all. For the same reason, a model with unions alone has predictions that are very similar to a model with unions and transfers. Tables 8 and 9 report the model results for the growth rate of output and the capital income share in Poland. I use gross domestic product per population ages $15-64$ that is detrended using an annual growth trend of 1.012. I use the same trend growth for Poland as for East Germany, in order to keep the two economies comparable.

The model with transfers and union power performs pretty well: it accounts for 83 percent of the growth in output for the period $1991-1996$, and roughly 40 percent of the growth in output over $1996-2004$. However, neither of the two experiments are able to capture the decline in output over the period 1989 to 1991. In the numerical examples performed for East Germany, a model with transfers and union power captured a big part of the decline in output due to a decline in the aggregate labor N. In the experiments performed for Poland, the decline in aggregate hours is not large enough to generate an output decline for the period

Table 8: Poland: Growth Rates of Output per Working-Age Person.

	Average annual growth over period (in %)		
	1989 − 1991	1991 − 1996	1996 − 2004
Data*	−10.0	3.0	2.2
Benchmark	13.0	4.1	1.0
Exp: Transfers and Union Power	13.3	2.5	0.9

*detrended using a 1.2 percent annual trend.

1989 to 1991. The experiment with transfers and union power will generate a small drop in output (0.2 percent) over the period 1991 to 1992 due to increase in union power in 1991. In the data, over the same period: 1991 to 1992, output per working-age person grew only by 0.3 percent. Hence model and data are close.

To better understand the period 1989 − 1991, other factors than the ones considered in the current theory should be evaluated. The current theory captures part of the decline in output per working-age person for East Germany, however it does not capture any of the corresponding decline for Poland. Atkeson and Kehoe (1993) provide a theory for understanding the recessions that followed the economic reforms in late 1980s.

Table 9 presents the results for the capital income share. Over the period 1991 − 2003, the capital income share varies slightly, but averages 0.33. Both models capture this movement very well.

Table 9: Poland: Capital Income Share of Output.

	1991	1996	1997 − 2003
Data	0.35	0.31	0.32
Benchmark	0.33	0.314	0.311
Exp: Transfers and Union Power	0.33	0.310	0.308

I conclude that a model with union power and transfers captures a slow growth in output per working-age person for Poland over the period 1991 − 1996, while being consistent with a fairly constant capital income share.

5.4 East Germany and Poland in Light of Theory

In this subsection, I compare the implications of the numerical experiments for East Germany and Poland.

First, I compare the benchmark model for East Germany to the benchmark model for Poland. The model experiment for Poland predicts higher initial growth of output than the benchmark model for East Germany. This is due to the fact that Poland starts with a much lower capital-output ratio than East Germany. Recall, the capital output ratio for Poland is 1.7 and 1989. For East Germany, the capital-output ratio in the benchmark model is 2.23 in 1989, so as match a capital-output ratio in data of 2.3 for year 1991.

Now, let's consider the model experiment with transfers and union power. The model predicts faster growth over the period $1991-1996$ for East Germany compared to Poland. Namely, the average annual growth in output over this period is 1.5 percentage points higher for East Germany than for Poland. The comparable figure in the data is 3.3 percentage points. The faster growth predicted by the model is due to the presence of high transfers for East Germany.

As emphasized in the previous sections the model also captures a roughly constant capital income share for Poland and a very low capital income share for East Germany in 1991. Key for this results is the model's technology, as well as the presence of policies. Low productivity plants have a capital income share lower than θ early during the transition. This reflects into an aggregate capital income share lower than θ.

I conclude that the model with union power and transfers quantitatively capture both the East German and Polish experience.

6 Conclusions

This paper built a theory of transition of an economy that gains access to a better, higher productivity technology. The theory was used to analyze the different transition experiences of two Central European economies after 1990. The main focus was on the growth experience, as well as the changes in the distribution of income between labor and capital. The experiences of East Germany and Poland are analyzed in greater detail. In terms of output per working-age person, East Germany experienced rapid initial growth from 1991 to 1996, followed by slow growth since mid 1990s; Poland experienced slower initial growth compared to East Germany, however this growth was sustained throughout the last decade. In terms of the share of income attributed to capital, East Germany had a dramatic increase in its very low capital income share between 1991 and 1996, followed by a leveling off at around 0.30 in early 2000s. Poland's capital income share during the transition period was fairly constant at 0.33.

The paper used a dynamic general equilibrium model to determine the driving forces that account for the different economic development of the two economies. I find that a large part of the differences in output growth over the period 1991 to 1996 can be accounted for by the presence of large tranfers to East Germany. Moreover, given the production technology and the presence of policies, the model is able to capture a very low initial capital income share for East Germany, and a roughly constant share for Poland. To be more specific, the model with transfers from the rest of the world and union power captures about 45 percent of the differences in output growth between East Germany and Poland over $1991-1996$. Moreover, the model captures a low capital income share of 22 percent for East Germany in 1991, while capturing a roughly constant capital share of 0.32 for Poland.

For future research, I consider addressing the output growth slowdown in East Germany since the mid 1990s. One interesting observation is that both East Germany and Poland displayed growth in output per working-age person of similar magnitude over the period $1991-2004$ (see Table 1). While most of the growth in output for East Germany took place in the early years of the transition, Poland displayed a relatively high growth in the last decade. This paper provides a theory for understanding the different growth development of the two economies at the beginning of the transition. In addressing the growth development since the mid 1990s, it is key to understand the reason behind the output growth slowdown in East Germany. To this end, attention should be focused on the driving forces behind the productivity slowdown in East Germany since 1996.

7 Appendix

The Appendix is organized as follows. The first section provides the proofs to some propositions in the main text. The second section provides the characterization of the jth union problem. Section 3 provides more detail on the policy implications for the capital income share of output. Section 4 presents sensitivity analysis for certain parameters. Section 5 contains the data appendix.

7.1 Proofs

Proof of Proposition 1

I derive the aggregate production function as the solution to program (1) presented in the text. There are five different cases. Each case corresponds to different constraints from (1) binding or not. In the derivations below, I assume $\alpha \equiv ((1-\theta) z_H / z_L)^{1/\theta} > 1$. For values of parameters such that $\alpha \leq 1$ the procedure employed to derive the production function is similar. The resulting aggregate production function differs only slighlty and is presented in the main text.

Case 1: All plants operate and are allocated more than the minimum labor requirement. That is, $m_i = M_i$ for both $i \in \{L, H\}$, and $n_i > \bar{n}$ for both $i \in \{L, H\}$.

The aggregate production function solves:

$$F(N, M_H, M_L) = \max_{\{n_i\}_{i\in\{H,L\}}} M_H z_H n_H^{1-\theta} + M_L z_L n_L^{1-\theta}$$

$$\text{s.t. } M_H n_H + M_L n_L = N$$

The first order conditions yield:

$$n_H = n_L \left(\frac{z_H}{z_L}\right)^{1/\theta}$$

Let $\rho \equiv \left(\frac{z_H}{z_L}\right)^{1/\theta}$. Then $n_H = \rho n_L$. Using the feasibility constraint one derives

$$n_L = \frac{N}{\rho M_H + M_L}$$

We now use the expressions for the optimal n_H and n_L to obtain the expression for F.

$$F(N, M_H, M_L) = z_L N^{1-\theta} (\rho M_H + M_L)^{\theta}$$

Recall we assumed $n_i > \bar{n}$. These constraints imply:

$$n_L = \frac{N}{\rho M_H + M_L} > \bar{n} \Leftrightarrow \frac{N}{\bar{n}} > \rho M_H + M_L$$

To summarize: If $N/\bar{n} > \rho M_H + M_L$, then $F(N, M_H, M_L) = z_L (\rho M_H + M_L)^{\theta} N^{1-\theta}$.

Case 2: All z_H type plants operate and are allocated at least the minimum labor requirement; only some z_L type plants operate, they operate at the minimum scale. That is, $m_H = M_H$, $0 < m_L < M_L$, $n_H \geq \bar{n}$, **and** $n_L = \bar{n}$.

The aggregate production function solves:

$$F(N, M_H, M_L) = \max_{n_H, m_L} M_H z_H n_H^{1-\theta} + m_L z_L \bar{n}^{1-\theta}$$
$$\text{s.t. } M_H n_H + m_L \bar{n} = N$$

The measure of low productivity plants that operate, m_L, is:

$$m_L = \frac{N - M_H n_H}{\bar{n}}$$

Then,

$$F(N, M_H, M_L) = \max_{n_H \geq \bar{n}} M_H z_H n_H^{1-\theta} + \left(\frac{N}{\bar{n}} - \frac{M_H n_H}{\bar{n}} \right) z_L \bar{n}^{1-\theta}$$

The first order condition yields:

$$n_H = \left(\frac{(1-\theta) z_H}{z_L} \right)^{1/\theta} \bar{n}$$

Let $\alpha \equiv \left(\frac{(1-\theta) z_H}{z_L} \right)^{1/\theta}$. If $\alpha > 1$, $n_H = \alpha \bar{n}$; otherwise $n_H = \bar{n}$. I focus here on $\alpha > 1$.

Recall we assumed $0 < m_L < M_L$, and $n_H \geq \bar{n}$. If $\alpha > 1$ then $n_H > \bar{n}$. Moreover,

$$0 < m_L < M_L \Leftrightarrow M_H\alpha < N/\bar{n} < \alpha M_H + M_L$$

To summarize: If $M_H\alpha < N/\bar{n} < M_H\alpha + M_L$ where $\alpha > 1$, then $F(N, M_H, M_L) = \left[z_L\bar{n}^{-\theta}\right] N + \left[\left(z_H\alpha^{1-\theta} - z_L\alpha\right)\bar{n}^{1-\theta}\right] M_H$.

Case 3. All plants operate; z_H type plants are allocated more than the minimum labor requirement, while z_L type plants operate at the minimum scale. That is, $m_i = M_i$ for both $i \in \{L, H\}$, $n_H > \bar{n}$, and $n_L = \bar{n}$.

The aggregate production function solves:

$$F(N, M_H, M_L) = \max_{n_H} M_H z_H n_H^{1-\theta} + M_L z_L \bar{n}^{1-\theta}$$
$$\text{s.t. } M_H n_H + M_L\bar{n} = N$$

Solution for n_H is:

$$n_H = \frac{N - M_L\bar{n}}{M_H}$$

The aggregate production function is given by:

$$F(N, M_H, M_L) = z_H M_H^{\theta}\left(N - M_L\bar{n}\right)^{1-\theta} + z_L\bar{n}^{1-\theta}M_L$$

This function correponds to values of factor inputs and parameters such that:

$$\alpha M_H + M_L < N/\bar{n} < \rho M_H + M_L \quad \text{where } \alpha > 1.$$

This follows from Case 2 and Case 1. Moreover, it is easy to check that $n_H > \bar{n}$.

Case 4. All z_H type plants operate and are allocated more than the minimum labor requirement; no z_L type plants operate. That is, $m_H = M_H$, $m_L = 0$, $n_H > \bar{n}$, and $n_L = 0$.

The aggregate production function solves:

$$F(N, M_H, M_L) = \max_{n_H} M_H z_H n_H^{1-\theta}$$
$$\text{s.t. } M_H n_H = N$$

The solution is $n_H = N/M_H$ and

$$F(N, M_H, M_L) = z_H M_H^{\theta} N^{1-\theta}$$

Recall we assumed $n_H > \bar{n}$. This implies that $N/M_H > \bar{n}$. Moreover, from Case 2. we know that if $M_H\alpha < N/\bar{n}$ the measure of low plants that operate, m_L will be strictly positive. Thus, if $N/\bar{n} < M_H\alpha$, $m_L = 0$.

To summarize: If $M_H < N/\bar{n} < \alpha M_H$, then $F(N, M_H, M_L) = z_H M_H^{\theta} N^{1-\theta}$.

Case 5. Only some z_H type plants operate, they operate at the minimum scale; no z_L type plants operate. That is, $m_H < M_H$, $m_L = 0$, $n_H = \bar{n}$, **and** $n_L = 0$.

The aggregate production function solves:

$$F(N, M_H, M_L) = \max_{m_H} m_H z_H \bar{n}^{1-\theta}$$
$$\text{s.t. } m_H \bar{n} = N$$

The solution is $m_H = N/\bar{n}$ and

$$F(N, M_H, M_L) = z_H \bar{n}^{-\theta} N$$

Recall we assumed $m_H < M_H$. This implies $N/\bar{n} < M_H$.

To summarize: If $N/\bar{n} < M_H$, then $F(N, M_H, M_L) = z_H \bar{n}^{-\theta} N$.

For parameter values such that $\alpha \leq 1$ I repeat the same procedure and derive the production function presented in the text.

This completes the proof. ■

Corollary 1. From the proof of Proposition 1. we have the following decision rules for plant labor:

$$n_H = \begin{cases} \bar{n} & \text{if } \eta_1 \leq N \\ N/M_H & \text{if } \eta_1 \leq N \leq \eta_2 \\ l_H & \text{if } \eta_2 \leq N \leq \eta_3 \\ (N - M_L\bar{n})/M_H & \text{if } \eta_3 \leq N \leq \eta_4 \\ \rho N/(\rho M_H + M_L) & \text{if } N \geq \eta_4 \end{cases}$$

where $l_H = \alpha\bar{n}$ if $\alpha > 1$ and $l_H = \bar{n}$ if $\alpha \leq 1$.

$$n_L = \begin{cases} 0 & \text{if } \eta_1 \leq N \\ 0 & \text{if } \eta_1 \leq N \leq \eta_2 \\ \bar{n} & \text{if } \eta_2 \leq N \leq \eta_3 \\ \bar{n} & \text{if } \eta_3 \leq N \leq \eta_4 \\ N/(\rho M_H + M_L) & \text{if } N \geq \eta_4 \end{cases}$$

where $\eta_i's$ are as given in the text of proposition 1.

Proof of Proposition 2

The proof has two parts. (i). First I show that when all high productivity plants operate, their marginal product is strictly higher than the marginal product of low productivity plants. (ii). Second, if there is an excess of high productivity plants, it is optimal to operate only some high productivity plants and no low productivity plants. In this case, the marginal product of all plants is zero.

To show (i) it suffices to prove that $\partial F/\partial M_H > \partial F/\partial M_L$ whenever $\partial F/\partial M_L$ is strictly positive (i.e. $N \geq \eta_3$). Using the aggregate production function, we have:

$$\frac{\partial F}{\partial M_H} = \begin{cases} \theta z_H M_H^{\theta-1} (N - M_L\bar{n})^{1-\theta} & \text{if } \eta_3 < N \leq \eta_4 \\ \\ \rho\theta z_L (\rho M_H + M_L)^{\theta-1} N^{1-\theta} & \text{if } N \geq \eta_4 \end{cases}$$

$$\frac{\partial F}{\partial M_L} = \begin{cases} z_L \bar{n}^{1-\theta} - \bar{n}\left(1-\theta\right) z_H M_H^{\theta} \left(N - M_L \bar{n}\right)^{-\theta} & \text{if } \eta_3 < N \leq \eta_4 \\ \\ \theta z_L \left(\rho M_H + M_L\right)^{\theta - 1} N^{1-\theta} & \text{if } N \geq \eta_4 \end{cases}$$

where as mentioned in the text, $\eta_4 = \left(\rho M_H + M_L\right) \bar{n}$ and η_3 is as below:

$$\begin{aligned} \text{If } \alpha &> 1, \text{then } \eta_3 = \left(\alpha M_H + M_L\right) \bar{n} \\ \text{If } \alpha &\leq 1, \text{then } \eta_3 = \left(M_H + M_L\right) \bar{n}. \end{aligned}$$

- If $N \geq \eta_4$, then $\partial F / \partial M_H > \partial F / \partial M_L$ follows immediately from the fact that $\rho \equiv \left(z_H / z_L\right)^{1/\theta} > 1$.

- If $\eta_3 < N \leq \eta_4$, proving $\partial F / \partial M_H > \partial F / \partial M_L$ is equivalent to proving:

$$\theta z_H M_H^{\theta - 1} \left(N - M_L \bar{n}\right)^{1-\theta} > z_L \bar{n}^{1-\theta} - \bar{n}\left(1-\theta\right) z_H M_H^{\theta} \left(N - M_L \bar{n}\right)^{-\theta}$$

Let's derive this inequality.

$$\begin{aligned} \xi &\equiv \theta z_H M_H^{\theta-1} \left(N - M_L \bar{n}\right)^{1-\theta} + \bar{n}\left(1-\theta\right) z_H M_H^{\theta} \left(N - M_L \bar{n}\right)^{-\theta} \qquad (13) \\ &= z_H M_H^{\theta-1} \left(N - M_L \bar{n}\right)^{-\theta} \left[\theta \left(N - M_L \bar{n}\right) + \bar{n}\left(1-\theta\right) M_H\right] \\ &= z_H M_H^{\theta-1} \left(N - M_L \bar{n}\right)^{-\theta} \left[\theta \left(N - \left(M_H + M_L\right) \bar{n}\right) + \bar{n} M_H\right] \end{aligned}$$

Two observations are useful:

$$N > \eta_3 \Leftrightarrow N > \left(M_H + M_L\right) \bar{n} \quad \text{for any } \alpha \qquad (14)$$

$$N \leq \eta_4 \Leftrightarrow N - M_L \bar{n} \leq \rho \bar{n} M_H \qquad (15)$$

Use (14) in (13) to obtain:

$$\begin{aligned} \xi &> z_H M_H^{\theta-1} \left(N - M_L \bar{n}\right)^{-\theta} \bar{n} M_H \\ &= z_H \bar{n} \left(\frac{M_H}{N - M_L \bar{n}}\right)^{\theta} \end{aligned}$$

Use (15) to obtain:

$$\begin{aligned}\xi &\geq z_H \bar{n} \left(\frac{1}{\rho \bar{n}}\right)^{\theta} \\ &= z_H \rho^{-\theta} \bar{n}^{1-\theta} \\ &= z_L \bar{n}^{1-\theta}\end{aligned}$$

where the last line results after substituting the expression for ρ.

- If $\eta_3 = N$, then the following hold:

  if $\alpha > 1$, then $\partial F/\partial M_L$ and $\partial F/\partial M_H > 0$, and

  if $\alpha \leq 1$, then

$$\begin{aligned}\partial F/\partial M_L &= (z_L - (1-\theta) z_H) \bar{n}^{1-\theta} > 0 \\ \partial F/\partial M_H &= \theta z_H \bar{n}^{1-\theta} > 0\end{aligned}$$

It is easy to see that $\partial F/\partial M_H > \partial F/\partial M_L$.

To see (ii) note that the production function is given by:

$$F(N, M_H, M_L) = z_H \bar{n}^{-\theta} N$$

In this case, all the share of total income attributed to capital is 0.

This completes the proof. ■

Proof of Proposition 5

In proving proposition 5., I first start with a Lemma.

Lemma 1. Consider an economy in which all high productivity plants operate at any given point in time. Let $\alpha \equiv ((1-\theta) z_H/z_L)^{1/\theta} > 1$. Then, the wage rate in this economy is such that:

(i). For all t, $w_t = (1-\theta) z_H n_{H,t}^{-\theta}$, and

(ii). For all t, $w_t \geq (1-\theta) z_L n_{L,t}^{-\theta}$, with equality if $n_L > \bar{n}$.

Proof. Each plant of type i maximizes profits subject to hiring at least $\bar{n}$ units of labor:

$$\pi_i \equiv \max_{n_i} z_i n_i^{1-\theta} - w n_i$$
$$\text{s.t. } n_i \geq \bar{n}$$

The first order condition for this problem yields:

$$w \geq (1-\theta) z_i n_i^{-\theta}, \quad \text{with equality if } n_i > \bar{n}.$$

In an economy in which all high productivity plants operate, the constraint on the minimum labor requirement will sometimes bind for the low productivity plants. Given $\alpha > 1$, $n_i \geq \bar{n}$ will not bind for the high productivity plants (See proof of Proposition 1. in Appendix 7.1 for details on the labor input decision rules). This completes the proof of the lemma. ■

The profits of a plant of type i are given by π_i. I make use of Lemma 1. and substitute for the wages in the expression for plant profits. Thus:

$$\begin{aligned}
\pi_H &= z_H n_H^{1-\theta} - \left[(1-\theta) z_H n_H^{-\theta}\right] n_H = \theta z_H n_H^{1-\theta} \\
\pi_L &\leq z_L n_L^{1-\theta} - \left[(1-\theta) z_L n_L^{-\theta}\right] n_L = \theta z_L n_L^{1-\theta}
\end{aligned}$$

Now we can calculate the capital income shares of each plant type:

$$\begin{aligned}
\phi_H &= \frac{\pi_H}{y_H} = \theta \\
\phi_L &= \frac{\pi_L}{y_L} \leq \theta, \quad \text{with equality if } n_i > \bar{n}.
\end{aligned}$$

This completes the proof of proposition 5. ■

7.2 Characterization of Equilibrium

Consider the jth union problem. Let $\lambda_t(j)$ denote the Lagrange multiplier on the budget constraint at time t. The first order conditions of this problem are summarized by the budget constraints, the laws of motion for capital, $X_{L,t}(j) = 0$, as well as:

$$\begin{aligned} \beta^t U_C [C_t(j), l_t(j)] &= \lambda_t(j) \cdot (1 + \tau_{C,t}) \\ \beta^t U_l [C_t(j), l_t(j)] \frac{\partial l_t(j)}{\partial w_t(j)} &= -\lambda_t(j) \cdot (1 - \tau_{N,t}) \left[l_t(j) + w_t(j) \frac{\partial l_t(j)}{\partial w_t(j)} \right] \\ \frac{\lambda_t(j)}{\lambda_{t+1}(j)} &= (1 + (1 - \tau_{M,t+1})(r_{H,t+1} - \delta_H)) \end{aligned} \tag{16}$$

and a transversality condition: $\lim_{t\to\infty} \beta^t U_C [C_t(j), l_t(j)] M_{H,t+1}(j) = 0$.

The Euler Equation is:

$$\frac{U_C(C_t(j), l_t(j))}{U_C(C_{t+1}(j), l_{t+1}(j))} = \beta \frac{1 + \tau_{C,t}}{1 + \tau_{C,t+1}} (1 + (1 - \tau_{M,t+1})(r_{H,t+1} - \delta_H))$$

In order to derive the intratemporal marginal rate of substitution between leisure and consumption I first derive $\partial l_t(j) / \partial w_t(j)$. Notice that making use of (5) twice, we can write:

$$\begin{aligned} \frac{\partial l_t(j)}{\partial w_t(j)} &= \frac{\partial \left[\left(\frac{w_t}{w_t(j)} \right)^{1/(1-\nu)} N_t \right]}{\partial w_t(j)} \\ &= w_t^{1/(1-\nu)} N_t \frac{1}{\nu - 1} w_t(j)^{1/(\nu-1)-1} \\ &= \frac{1}{\nu - 1} \frac{l_t(j)}{w_t(j)} \end{aligned}$$

Thus, equation (16) becomes:

$$\beta^t U_l [C_t(j), l_t(j)] \frac{1}{\nu - 1} \frac{l_t(j)}{w_t(j)} = -\lambda_t (1 - \tau_{N,t}) \frac{\nu}{\nu - 1} l_t(j)$$
$$\Leftrightarrow \beta^t U_l [C_t(j), l_t(j)] = -\lambda_t (1 - \tau_{N,t}) \nu w_t(j)$$

The intratemporal condition may be written as:

$$w_t(j) = -\frac{1}{\nu} \frac{(1 + \tau_{C,t})}{(1 - \tau_{N,t})} \frac{U_l [C_t(j), l_t(j)]}{U_C [C_t(j), l_t(j)]} \tag{17}$$

7.3 Policy Implications for capital income share

In Section 4.4, I presented four experiments to point out that policies may affect the capital income share in a nonlinear fashion. The focus was on a snapshot in time, namely the capital income share at time $t = 3$ in the model. Here, I present details on two of the experiments considered in the text: the experiment with the transfer policy alone, and the experiment with union power alone. I use the parameter values as indicated in Table 10.

Table 10: Impact of Policies: Numerical Example Parameters.

$\beta = 0.96$	$\tau_C = 0$
$\delta_H = \delta_L = 0.06$	$\tau_N = 0$
$\theta = 0.33$	$(M_H + M_L)/Y = 2.8$
$\psi = 4.15$	$M_H/(M_H + M_L) = 0.07$
$z_H = 12, \quad z_L = 7.5$	$\bar{n} = 0.004$

Figure 6 presents the model equilibrium paths for the capital income share, for every value of the policy. The top left-hand panel of Figure 6 presents the capital income share in the model with $\overline{Tr} = 0\%$ and $\overline{Tr} = 60\%$ of GDP. In both examples, the capital income share starts at $\theta = 0.33$, it declines and then increases back towards θ when the low productivity plants operate at $\bar{n}$. Finally, when there are only high productivity plants operating, the capital share is again θ. One observation is that higher transfers lead to a bigger "dip" in the capital income share. The same result occurs in the experiment with union power as well. The lower panels of Figure 6 present the capital share for all the values of the policies.

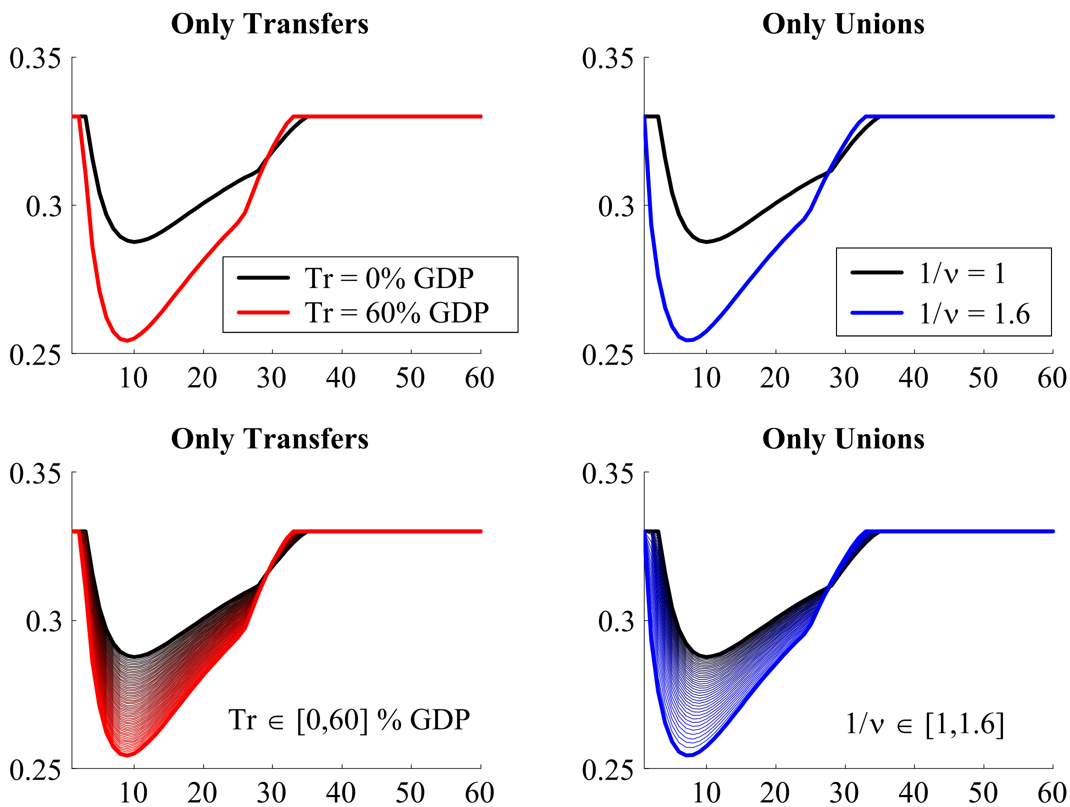
7.4 Sensitivity Analysis

In this section, I report results for different values of the z_H/z_L ratio. Moreover, I report the model's implications in the case in which union power, $1/\nu_t$ declines over time starting year 1991.

- Different capital productivities ratios (i.e. z_H/z_L)

I consider the experiment with both the transfer and the union power policy for East Germany and Poland. I report the results for output growth and the capital income share for

Figure 6: Capital Income Share in Model with Policies



3 values of z_H/z_L (see Tables 11 and 12). All other parameters are as specified in the main text. Hence, the results for $z_H/z_L = 1.53$ are the same as the ones presented in the text.

Let me interpret the results. A higher ratio z_H/z_L means that the economy is transitioning towards a higher productivity capital. The investment in the high productivity capital is higher in economies with higher z_H/z_L. Hence, the transition is faster. This means, that low productivity plants shut down faster.

A faster accumulation of the z_H type plants, leads to faster growth over the period $1991-1996$, as shown in Table 11. The impact on the capital share for the period $1991-1996$ is very small. A greater impact is observed for the period $1997-2003$. This is due to z_L type plants becoming idle faster. That is, when only some low productivity plants operate, the capital share is increasing at a steep rate. This was presented in Figure 4.

Note that experiments with higher values of z_H/z_L predict better results for output growth. Higher values of z_H/z_L yield a faster initial growth in East Germany compared to

Table 11: Growth Rates of Output per Working-Age Person.

	Average annual growth over period (in %)		
	1989 – 1991	1991 – 1996	1996 – 2004
East Germany			
$z_H/z_L = 1.53$	−8.5	4.0	2.3
$z_H/z_L = 1.55$	−8.2	4.3	1.7
$z_H/z_L = 1.57$	−7.9	4.8	1.5
Poland			
$z_H/z_L = 1.53$	13.6	2.5	0.9
$z_H/z_L = 1.55$	13.8	2.7	0.9
$z_H/z_L = 1.57$	14.2	2.8	1.0

Table 12: Capital Income Share of Output.

	1991	1996	1997 – 2003
East Germany			
$z_H/z_L = 1.53$	0.223	0.227	0.250
$z_H/z_L = 1.55$	0.225	0.229	0.267
$z_H/z_L = 1.57$	0.227	0.230	0.286
Poland			
$z_H/z_L = 1.53$	0.33	0.315	0.312
$z_H/z_L = 1.55$	0.33	0.309	0.307
$z_H/z_L = 1.57$	0.33	0.307	0.305

Poland. However, these experiments predict a counterfactual rapid increase in the capital share during 1997 – 2003, while preserving a similar behavior over the 1991 – 1996 period. This leads me to conclude that lower values of z_H/z_L are more reasonable.

- Union power declining over time

I consider the experiment with both the transfer and the union power policy for East Germany. The results with constant union power after 1991 are same as the ones presented in the main text. I report the results on output growth and the capital income share for a declining union power starting model year 1991. I choose a constant rate of decline.

Table 13: East Germany: Model Results If Union Power Declines Over Time

	Average annual growth over period (in %)		
OUTPUT	1989 – 1991	1991 – 1996	1996 – 2004
Union power is constant after 1991	−8.5	4.0	2.3
Union power declines after 1991	−8.9	4.5	2.7
CAPITAL INCOME SHARE	1991	1996	1997 – 2003
Union power is constant after 1991	0.22	0.23	0.25
Union power declines after 1991	0.22	0.24	0.26

If union power declines over time following 1991, the model predicts faster growth over the 1991 – 1996 period. Moreover, the capital income share predictions are slighlty improved, in the sense that the capital share increases more over the period 1991 – 2003.

Although predictions seem more in line with data, the faster growth in output observed in a model with declining union power is due to large increases in hours worked. These increases are counterfactual. In data, over the period 1991 – 2003 average hours worked, N, declined by 2.8 hours per week. The model with constant union power after 1991, predicts a slight increase of 1 hour, over the same period. However, the experiment with union power declining over time after 1991, predicts an increase of 3 hours.

7.5 Data Appendix

Some data sources were used for obtaining multiple series. I use the following abreviations to refer to them.

GGDC: Groningen Growth and Development Centre and the Conference Board, Total Economy Database, May 2006, http://www.ggdc.net.

VGRdL: Statistisches Aemter Der Bundes und Der Laender, Volkswirtschaftliche Gesamtrechnungen der Länder, http://www.statistik-bw.de/Arbeitskreis_VGR/

SourceOECD: Organization for Economic Co-operation and Development, www.sourceoecd.org.

Destatis: Federal Statistical Office of Germany, http://www.destatis.de/e_home.htm, GENESIS-Online Database.

The following data sources were used to calculate gross domestic product per working-age person.

Gross Domestic Product

East Germany: GGDC, VGRdL.

Czech Republic, Slovak Republic, Poland: GGDC.

Population $15 - 64$

East Germany:

- Destatis, Series $12411 - 0011$, Bevoelkerung: Bundeslaender, Stichtag, Altersjahre.
- Statistisches Jahrbuch Der Deutschen Demokratischen Republik, Staatsdrukerei der Deutschen Demokratischen Republik, 1990.

Czech Republic, Slovak Republic, Poland:

- SourceOECD Employment and Labour Market Statistics.

Capital Income Share of Output

The following data sources were used to calculate the capital income share of output.

East Germany: Proprietor's income is estimated using data on number of self-employed, and the assumption that they earn the same labor income of employees.

- VGRdL, Excel Table: Production, expenditure and distribution of the gross domestic product in Germany by Bundesland and East-West-Regions, 1991 to 2004, and other.
- Destatis, Series $13311 - 0002$ Erwerbstätigenrechnung des Bundes und der Laender.

Czech Republic, Slovak Republic, Poland:

- SourceOECD National Accounts Database, Annual National Accounts, Volume 1, Main Aggregates and Volume 2, Detailed Tables.

Hourly Wage Rate

Hourly wage rates are calculated as hourly compensation of employees deflated using the consumption deflator. The following data were used:

East Germany: VGRdL

Poland:

- GGDC
- SourceOECD, Employment and Labour Market Statistics, Series: Employment by Professional Status.
- SourceOECD National Accounts Database, Annual National Accounts, Volume 1, Main Aggregates.

Transfers from the Rest of the World

Data on transfers were obtained from the following sources.

East Germany:

- European Commision, "Germany's Growth Performance in the 1990's," Directorate-General for Economic and Financial Affairs, Economic Papers No. 170, May 2002.
- Jensen, Heinz "Transfers to Germany's eastern Laender: a necessary price for convergence of a permanent drag?," European Commision, Directorate-General for Economic and Financial Affairs, ECFIN Country Focus, Volume 1, Issue 16, 2004.

Poland:

- OECD, Poland, OECD Economic Surveys, 2004, no. 8, pp. 1-215.
- OECD, Poland, OECD Economic Surveys, 2006, no. 11, pp. 1-145.
- Hallet, Martin and Filip Keereman "Budgetary transfers between the EU and the new Member States: manna from Brussels or a fiscal drag?," European Commision, Directorate-General for Economic and Financial Affairs, ECFIN Country Focus, Volume 2, Issue 2, 2005.

Income Side Accounts for Year 1989, East Germany

- Sinn and Sinn (1992)
- United Nations, National Accounts Statistics: Main Aggregates and Detailed Tables, Part I, (1988)
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